Electron Spin

$$\hat{H} = \frac{\vec{p}^{\,2}}{2m^*} + \mu_B H \sigma \Rightarrow \mathcal{E}(k,\sigma) = \frac{\hbar^2 k^2}{2m^*} + \mu_B H \sigma$$

Here $\mu_B = \frac{e\hbar}{2m_c c}$ = Bohr magneton $= 5.79 \times 10^{-9} \frac{eV}{G}$

At $T=0$ there are separate up and down spin Fermi surfaces:

$$\mathcal{E}_F = \frac{\hbar^2 k_{F+}^2}{2m^*} + \mu_B H = \frac{\hbar^2 k_{F-}^2}{2m^*} - \mu_B H$$

$$n_+ = \frac{k_{F+}^3}{6\pi^2}, \quad n_{F-} = \frac{k_{F-}^3}{6\pi^2}$$

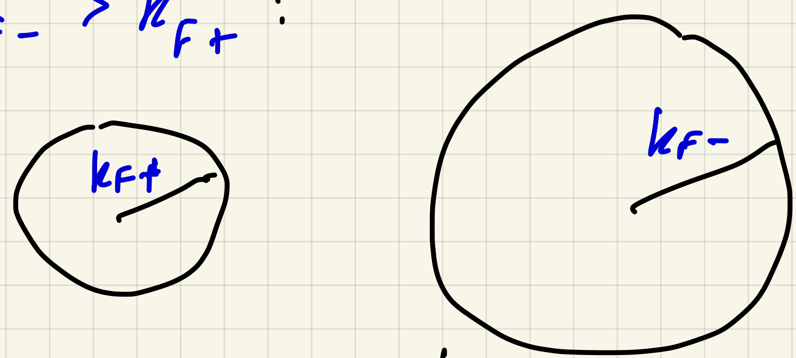
Thus, 2 eqns in 2 unknowns $k_{F\pm}$ in terms of n and H:

$$6\pi^2 n = k_{F+}^3 + k_{F-}^3$$

$$\frac{4m^*\mu_B}{\hbar^2} H = \left(\frac{m^*}{m_e}\right) \cdot \left(\frac{2e}{\hbar c}\right) \cdot H = k_{F-}^2 - k_{F+}^2$$

m^* is the <u>effective mass</u>, which takes into account effects of the crystalline ionic lattice in solids. Typically in semiconductors one has $m^* \approx \frac{1}{10} m_e$.

Thus, $k_{F-} > k_{F+}$:



Setting $k_{F+} = 0$, we obtain the critical field at which 100% polarization sets in:

$$H_c = \frac{m_e}{m^*} \cdot \frac{\hbar c}{2e} \cdot (6\pi^2 n)^{2/3}$$

Pauli paramagnetism

$$\varepsilon_\sigma(\vec{k}) = \frac{\hbar^2 k^2}{2m^*} + \mu_B H \sigma \quad ; \quad \mu_B = \frac{e\hbar}{2m_e c}$$

Number density:

$$n = \int d\varepsilon\, g_\uparrow(\varepsilon) f(\varepsilon - \mu) + \int d\varepsilon\, g_\downarrow(\varepsilon) f(\varepsilon - \mu)$$

with $g_\sigma(\varepsilon) = \frac{1}{2} g(\varepsilon - \sigma \mu_B H)$

$$n = \int_{-\infty}^{\varepsilon_F} d\varepsilon\, g(\varepsilon) + g(\varepsilon_F)\delta\mu + \frac{\pi^2}{6}(k_B T)^2 g'(\varepsilon_F) + (\mu_B H)^2 g'(\varepsilon_F) + \dots$$

Then

$$\delta\mu(T,n,H) = -\left\{\frac{\pi^2}{6}(k_BT)^2 + (\mu_BH)^2\right\}\frac{g'(\varepsilon_F)}{g(\varepsilon_F)} + \ldots$$

and

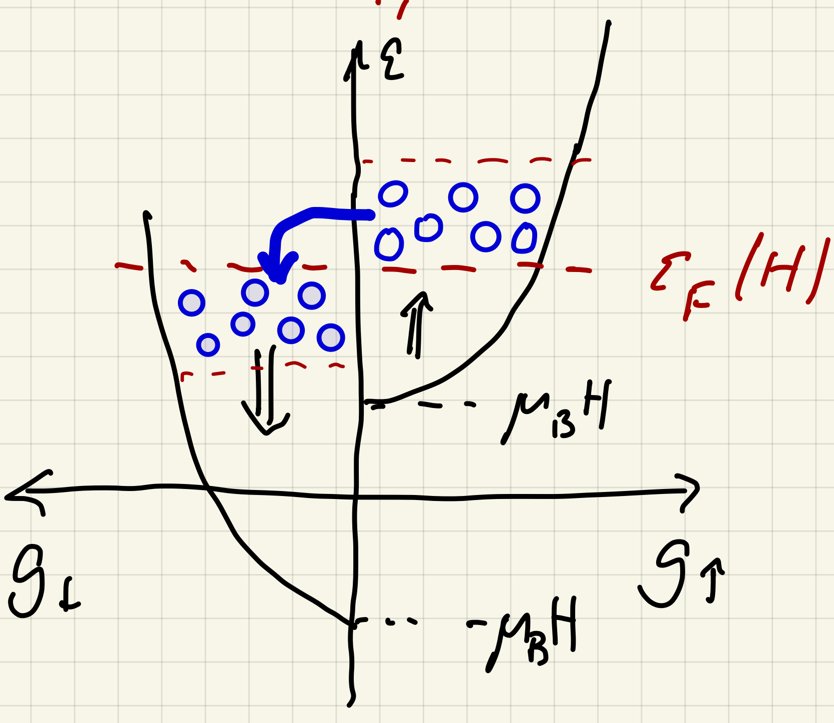
$$n_\uparrow - n_\downarrow = \int d\varepsilon\left[g_\uparrow(\varepsilon) - g_\downarrow(\varepsilon)\right]f(\varepsilon-\mu)$$

$$= -\mu_BH\ \pi D \csc\pi D\, g(\mu) + \mathcal{O}(H^3)$$

$$\Rightarrow\ M = -\mu_B(n_\uparrow - n_\downarrow) = \mu_B^2\, g(\varepsilon_F)H$$

Pauli paramagnetic susceptibility:

$$\chi = \left.\frac{\partial M}{\partial H}\right|_{T,n} = \mu_B^2\, g(\varepsilon_F)$$



Landau diamagnetism

The Zeeman Hamiltonian $H_z = \mu_B H \sigma$ is one of two ways electrons (or any charged particles) know about magnetic fields. The second way is through the electromagnetic vector potential $\vec{A}(\vec{r})$.

The full QM Hamiltonian for an electron in an external magnetic field is

$$\hat{H} = \frac{1}{2m^*}\left(\vec{p} + \frac{e}{c}\vec{A}\right)^2 + \mu_B H \sigma$$

m^* is the "effective mass" in solids

where $\vec{\nabla} \times \vec{A} = \vec{H}$. In QM class we learn that the energy eigenvalues are

$$\varepsilon(n, k_z, \sigma) = (n+\tfrac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*} + \sigma\mu_B H$$

where $\omega_c = \frac{eH}{m^* c}$ = cyclotron frequency

and $\hbar\omega_c$ = cyclotron energy. Then

$(n+\frac{1}{2})\hbar\omega_c$ = energy of n^{th} Landau level $(n = 0, 1, \ldots)$

Note $\frac{\mu_B H}{\hbar\omega_c} = \frac{m^*}{2m_e} = \frac{\text{Zeeman energy}}{\text{cyclotron energy}}$

In semiconductors, typically $m^* \sim 0.1\, m_e$

GaAs : $m^* = 0.067\, m_e$, $\hbar\omega_c / k_B \simeq 20\,K \cdot H[T]$

Then $\varepsilon(n, k_z, \sigma) = (n + \frac{1}{2} + \frac{1}{2} r\sigma)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m^*}$

with $r \equiv \frac{m^*}{m}$. In general, m^* is a symmetric tensor, i.e. $m^*_{\alpha\beta}$. Here we assume $m^*_{\alpha\beta} = m^* \delta_{\alpha\beta}$.

The grand potential is then

$$\Omega(T, V, \mu, H) = -\frac{V}{\lambda_T^2}\hbar\omega_c \sum_{n=0}^{\infty} \sum_{\sigma=\pm} Q\left((n + \frac{r\sigma+1}{2})\hbar\omega_c - \mu\right)$$

with

$$Q(\nu) = -\frac{V}{\lambda_T^2} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \log\left[1 + e^{-\nu/k_B T} e^{-\hbar^2 k_z^2 / 2m^* k_B T}\right]$$

$$= \frac{V}{\lambda_T^3} \mathrm{Li}_{3/2}\left(e^{-\nu/k_B T}\right)$$

Now use Euler-MacLaurin :

$$\sum_{n=0}^{\infty} F(n) = \int_0^{\infty} dx\, F(x) + \frac{1}{2} F(0) - \frac{1}{12} F'(0) + \dots$$

to obtain

$$\Omega(T,V,\mu,H) = \sum_{\sigma=\pm} \int_{-\infty}^{\infty} d\varepsilon\, Q(\varepsilon-\mu)\, \Theta\left(\varepsilon - \tfrac{1}{2}(1+\sigma r)\hbar\omega_c\right)$$

$$+ \frac{1}{2}\hbar\omega_c \sum_{\sigma} \left[Q\left(\frac{1+\sigma r}{2}\hbar\omega_c - \mu\right) - \frac{\hbar\omega_c}{6} Q'\left(\frac{1+\sigma r}{2}\hbar\omega_c - \mu\right) + \dots \right]$$

$$= 2\int_0^{\infty} d\varepsilon\, Q(\varepsilon-\mu) + \left(\frac{1}{4}r^2 - \frac{1}{12}\right)(\hbar\omega_c)^2 Q'(-\mu) + \dots$$

as an expansion in powers of H. Then

$$\chi = -\frac{1}{V}\frac{\partial^2\Omega}{\partial H^2} = \left(1 - \frac{m_e^2}{3m^{*2}}\right)\mu_B^2\, n^2 K_T$$

↑ compressibility

$$K_T = n^{-2}\frac{\partial n}{\partial \mu} \quad ; \quad n = \int_{-\infty}^{\mu} d\varepsilon\, g(\varepsilon) + \frac{\pi^2}{6}(k_B T)^2 g'(\mu) + \dots$$

$$K_T = n^{-2} g(\mu) + \frac{\pi^2}{6}\left(\frac{k_B T}{n}\right)^2 g''(\mu)$$

So for $T \ll T_F$, $K_T = n^{-2} g(\varepsilon_F) = \dfrac{d}{2n\varepsilon_F(n)} \propto n^{-(d+2)/d}$

$m^* < \dfrac{m_e}{\sqrt{3}} \Rightarrow \chi < 0$ (diamagnetic) and $m^* > \dfrac{m_e}{\sqrt{3}}$
$\Rightarrow \chi > 0$ (paramagnetic)