$$\Rightarrow \mu = -k_B T \log\left(1 + \frac{1}{V n_0}\right)$$

n_0 finite $\Rightarrow \mu = -k_B T / V n_0 \to 0^-$

<u>Summary</u>

- Critical temperature $T_c(n)$:

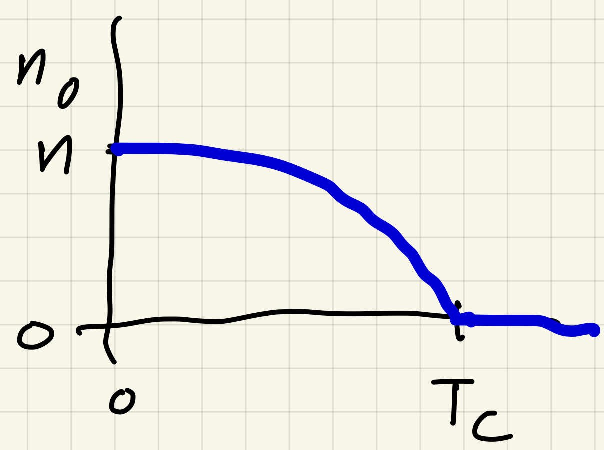
$$n = \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{\varepsilon/k_B T_c} - 1}$$

- $T < T_c(n)$:

$$n = n_0 + \overbrace{\int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{\varepsilon/k_B T} - 1}}^{n'} , \quad \mu = 0^-$$
$$z = 1^-$$

At $T = T_c$, $n_0 = 0$

At $T = 0$, $n_0 = n$



Independent intensive variables: T, n_0

Dependent ones : $n = n(T, n_0)$

- $T > T_c(n)$:

$$n = n(T,\mu) = \int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{(\varepsilon-\mu)/k_B T} - 1}$$

$n_0 = 0$, $z < 1$

- Ballistic dispersion : $\varepsilon(\vec{k}) = \frac{\hbar^2 k^2}{2m}$, $d = 3$

$$n(T,z) = \lambda_T^{-3} \, Li_{3/2}(z) \qquad (T > T_c)$$

where $Li_s(z) = \sum_{j=1}^{\infty} \frac{z^j}{j^s}$ polylogarithm function

$$= z + \frac{z^2}{2^s} + \frac{z^3}{3^s} + \cdots$$

$$p(T,z) = k_B T \, \lambda_T^{-3} \, Li_{5/2}(z) \qquad (T > T_c)$$

$T < T_c$:

$$n = n_0 + \zeta\left(\tfrac{3}{2}\right) \lambda_T^{-3} \qquad (T < T_c)$$

$$p = \zeta\left(\tfrac{5}{2}\right) k_B T \, \lambda_T^{-3}$$

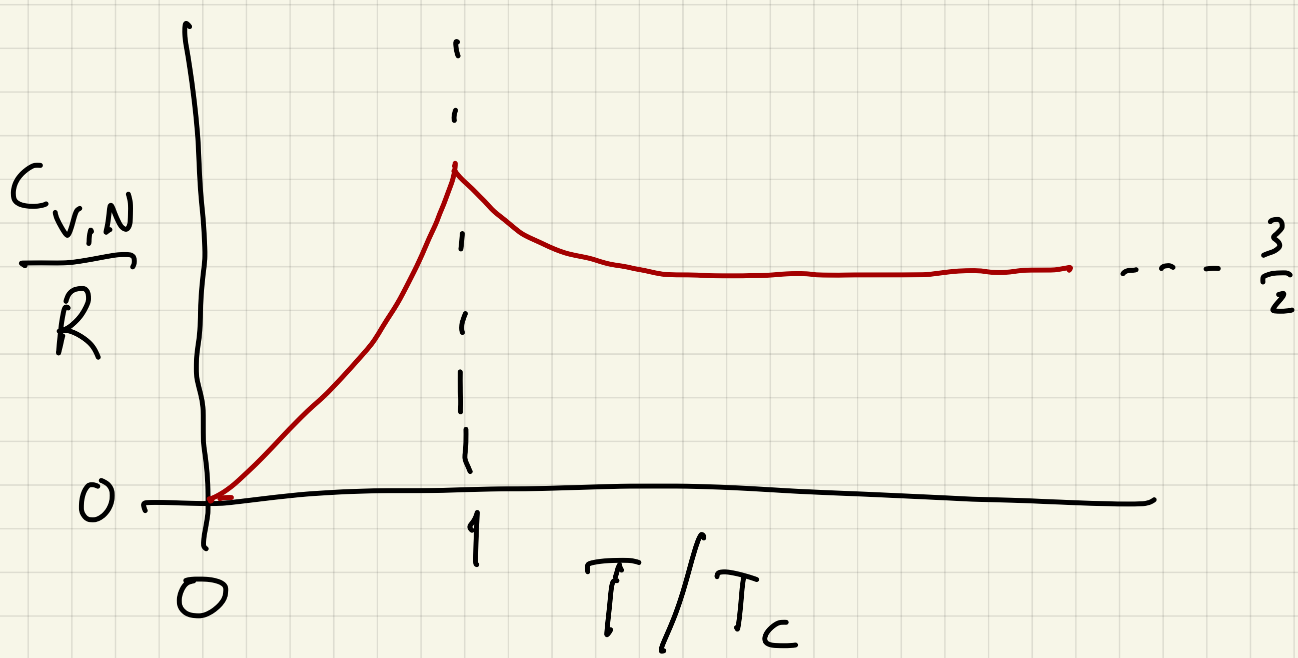
$$\mathrm{Li}_s(z) = \sum_{j=1}^{\infty} \frac{z^j}{j^s} \Rightarrow \mathrm{Li}_s(1) = \sum_{j=1}^{\infty} \frac{1}{j^s}$$

$$= \zeta(s)$$

$$T = T_c : n_0 = 0 \Rightarrow n = \zeta\left(\tfrac{3}{2}\right) \lambda_{T_c}^{-3}$$

I.e. $n \lambda_{T_c}^3 = \zeta(3/2)$

$$k_B T_c = \frac{2\pi \hbar^2}{m} \left(\frac{n}{\zeta(3/2)} \right)^{2/3} \propto n^{2/3}$$

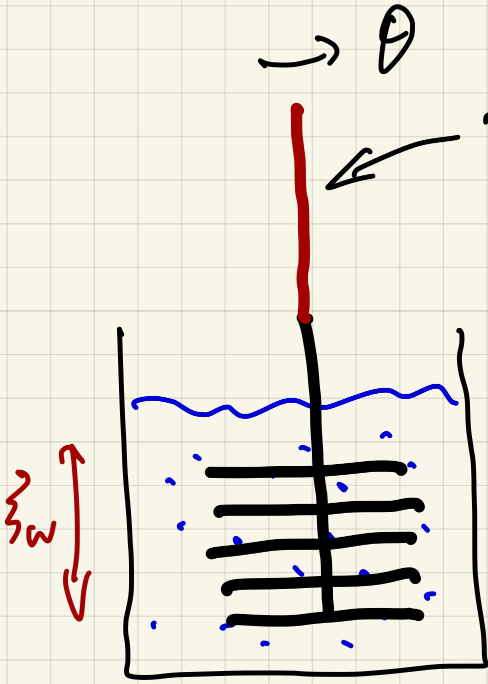


Andronikashvili (1946) expt

SF condensate has zero viscosity!

$$T < T_c : \quad n(T) = n_0 + \underbrace{n'(T)}_{\int_0^\infty d\varepsilon \, \frac{g(\varepsilon)}{e^{\beta\varepsilon} - 1}}$$

$$\xi_\omega \propto \sqrt{\nu/\omega}$$



torsional fiber

$$\tau = -K\theta$$

$$I\ddot{\theta} = -K\theta$$

$$\ddot{\theta} = -\omega^2\theta$$

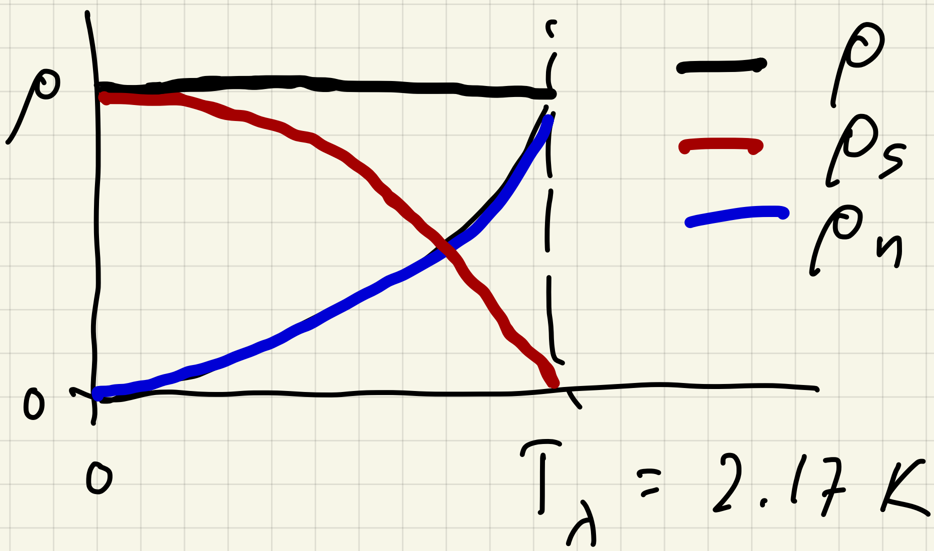
$$\omega = \sqrt{K/I}$$

$$T_{osc} = 2\pi/\omega$$

$$I = I_{app} + I_{He}(T)$$

$$I_{He}(T) = I_{He}(T_\lambda) \times \frac{\rho_n(T)}{\rho}$$

$$\rho = \rho_n + \rho_s$$



Difference between $\frac{\rho_s}{\rho}$ = SF fraction

and $N_{\vec{k}=0}/N$ = condensate fraction

SF wavefunction (many-body):

$$\Psi(\vec{r}_1, \ldots, \vec{r}_N) = \prod_{i<j}^{N} f(\vec{r}_i - \vec{r}_j)$$ "Jastrow state"

overlap of ${}^4He \Rightarrow |\Psi|^2 \simeq 0$