PHYSICS 140B W26 : STATISTICAL PHYSICS
HW ASSIGNMENT #7

(1) Consider the collisionless Boltzmann equation for the Hamiltonian $\hat{H}(p) = \frac{1}{4}Ap^4$ in one space dimension. Suppose the initial distribution is given by

$$f(x,p,t=0) = C\, e^{-x^2/2\sigma^2}\, e^{-p^2/2\kappa^2} \quad .$$

(a) Find $f(x,p,t)$ for all $t > 0$.

(b) Find the equation for the locus of points (x,p) for which $f(x,p,t) = C\,\exp(-\alpha^2/2)$.

(c) Express your result in (b) in dimensionless form and plot it for various values of the dimensionless time.

(2) Consider an ideal gas of point particles in $d = 3$ dimensions with isotropic dispersion $\varepsilon(\boldsymbol{p}) = A|\boldsymbol{p}|^4$.

(a) Find the enthalpy per particle $h = \mu + Ts$, where μ is the chemical potential and s is the entropy per particle. (You may find it useful to review some of the material in chapter 4 of the notes.)

(b) Find the thermal conductivity κ within the relaxation time approximation.

(3) Consider a nonequilibrium distribution of the form

$$f(\boldsymbol{r},\boldsymbol{p},t=0) = h^{-3} n \lambda_T^3\, e^{-\boldsymbol{p}^2/2mk_{\rm B}T}\left(1 + \frac{\alpha\,\boldsymbol{p}^2}{2mk_{\rm B}T}\right)$$

and investigate its relaxation to the equilibrium distribution $f^0(\boldsymbol{p}) = h^{-3} n \lambda_T^3\, e^{-\boldsymbol{p}^2/2mk_{\rm B}T}$ using the Boltzmann equation in the relaxation time approximation, with no external forces. Find $f(\boldsymbol{r},\boldsymbol{p},t)$. Then find $N(t)$ and $E(t)$, the time-dependent values for the total particle number and total energy. You may abbreviate $N_0 \equiv nV$, where V is the system volume and N_0 is the number of particles at equilibrium. Then, drawing upon your understanding of collisional invariants, explain why your calculation is complete BS. What has gone wrong?