PHYSICS 140B W26 : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) For a system of noninteracting $S = 0$ bosons obeying the dispersion $\varepsilon(\boldsymbol{k}) = \hbar v|\boldsymbol{k}|$.

(a) Find the density of states per unit volume $g(\varepsilon)$ in d space dimensions.

(b) Determine the critical temperature for Bose-Einstein condensation in $d = 3$ dimensions.

(c) Find the condensate fraction n_0/n for $T < T_c$.

(d) For this dispersion, is there a finite transition temperature in $d = 2$ dimensions? If not, explain why. If so, compute $T_c^{(d=2)}$.

(2) Using the argument we used in class and in §5.4.3 of the notes, predict the surface temperatures of the remaining planets in our solar system. In each case, compare your answers with the results below. In cases where there are discrepancies, try to come up with a convincing excuse.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
a (10^8 km)	0.576	1.08	1.50	2.28	7.78	14.3	28.7	45.0	59.1
$T_{\text{surf}}^{\text{obs}}$ (K)	340^*	$735^\dagger$	$288^\ddagger$	210	112	84	53	55	44

Table 1: Relevant planetary data. Observed temperatures are averages. *mean equatorial temperature, †mean temperature below cloud cover. **I say Pluto is a planet.**

(3) In §5.4.4 of the lecture notes we derived the spectral energy density $\rho_\varepsilon(\nu, T)$ for a three-dimensional blackbody. We found that it was peaked at a frequency $\nu^* = s^* k_B T/h$ where $s^* = 2.83144$ extremizes the function $s^3/(e^s - 1)$. Consider instead the function $\tilde{\rho}_\varepsilon(\lambda, T)$ as a function of wavelength λ and temperature T, where $\lambda = c/\nu$. To relate $\rho_\varepsilon(\nu, T)$ and $\tilde{\rho}_\varepsilon(\lambda, T)$, set the fraction of energy of EM radiation between frequencies ν and $\nu + d\nu$ equal to the fraction of energy between wavelengths λ and $\lambda + d\lambda$. Show that this is maximized at a wavelength $\lambda^* = t^* hc/k_B T$, where t^* is a constant. Find t^* numerically. Is $t^* = 1/s^*$? Why or why not?

(4) A nonrelativistic Bose gas consists of particles of spin $S = 1$. Each boson has mass m and magnetic moment μ_0. A gas of these particles is placed in an external field H.

(a) What is the relationship of the Bose condensation temperature $T_c(H)$ to $T_c(H = 0)$ when $\mu_0 H \gg k_B T$?

(b) Find the magnetization M for $T < T_{\mathrm{c}}$ when $\mu_0 H \gg k_{\mathrm{B}} T$. Calculate through order $\exp(-\mu_0 H / k_{\mathrm{B}} T)$.