PHYSICS 140B W26 : STATISTICAL PHYSICS
HW ASSIGNMENT #1

(1) You know that at most one fermion may occupy any given single-particle state. A *parafermion* is a particle for which the maximum occupancy of any given single-particle state is k, where k is an integer greater than zero. (For $k = 1$, parafermions are regular everyday fermions; for $k = \infty$, parafermions are regular everyday bosons.) Consider a system with one single-particle level whose energy is ε, *i.e.* the Hamiltonian is simply $\mathcal{H} = \varepsilon n$, where n is the particle number.

(a) Compute the partition function $\Xi(\mu, T)$ in the grand canonical ensemble for parafermions.

(b) Compute the occupation function $n_\varepsilon(\mu, T)$. What is n when $\varepsilon = -\infty$? When $\varepsilon = \mu$? When $\varepsilon = +\infty$? Does this make sense? Show that $n_\varepsilon(\mu, T)$ reduces to the Fermi and Bose distributions in the appropriate limits.

(c) Sketch $n_\varepsilon(\mu, T)$ as a function of μ for both $T = 0$ and $T > 0$ when $k = 3$.

(2) A gas of quantum particles with photon statistics has dispersion $\varepsilon(\boldsymbol{k}) = A\,|\boldsymbol{k}|^4$.

(a) Find the single particle density of states per unit volume $g(\varepsilon)$.

(b) Repeat the arguments in ch. 5 of the lecture notes for this dispersion.

(c) Assuming our known values for the surface temperature of the sun, the radius of the earth-sun orbit, and the radius of the earth, what would you expect the surface temperature of the earth to be if the sun radiated particles with this dispersion instead of photons?

(3) A three-dimensional gas of particles obeying photon statistics has the dispersion $\varepsilon(\boldsymbol{k}) = A\,k^{5/2}$. There are no internal degrees of freedom (*i.e.* the degeneracy factor is $\mathsf{g} = 1$). The number density is n.

(a) Compute the single particle density of states $g(\varepsilon)$.

(b) Compute the temperature $T(n)$.

(c) Compute the entropy density $s(n) = S/V$.

(d) For bosons and fermions, compute the second virial coefficient $B_2(T)$.

(4) Consider a three-dimensional ultrarelativistic gas, with dispersion $\varepsilon = \hbar c|\boldsymbol{k}|$. Find the viral expansion of the equation of state $p = p(n, T)$ to order n^3 for both bosons and fermions. You may find the `Mathematica` function `InverseSeries` quite useful in this regard.