Problem 1
A source produces a sound wave in air with frequency 1kHz. The maximum excess pressure is 0.03Pa. Air density is 1.20 kg/m3 and speed of the wave is 344m/s. Find:
(a) maximum displacement of air molecules s_max
(b) maximum excess density
(c) maximum velocity of molecules associated with the displacement

(a) Maximum displacement (s_{max})
Using the formula $P_{max} = \rho v \omega s_{max}$, where $\omega = 2\pi f$:

$$s_{max} = \frac{P_{max}}{2\pi f \rho v}$$

$$s_{max} = \frac{0.03}{2\pi(1000)(1.20)(344)}$$

$s_{max} \approx 1.16 \times 10^{-8}\ \mathrm{m}$

(b) Maximum excess density ($\Delta\rho_{max}$)
Using the relationship between pressure and density $\Delta P = v^2 \Delta\rho$:

$$\Delta\rho_{max} = \frac{P_{max}}{v^2}$$

$$\Delta\rho_{max} = \frac{0.03}{344^2}$$

$\Delta\rho_{max} \approx 2.54 \times 10^{-7}$ kg/m^3

(c) Maximum velocity (v_{max})

The maximum particle velocity is $v_{max} = \omega s_{max}$:

$$v_{max} = 2\pi(1000)(1.155 \times 10^{-8})$$

$v_{max} \approx 7.27 \times 10^{-5}$ m/s

Problem 2
(a) At a certain location at the instant where the excess pressure is 0.015Pa and
increasing, what is the displacement at that location?
(b) And if instead the pressure is decreasing?
(c) What is the excess density or deficit in density for cases (a) and (b)?

(a) Displacement when pressure is 0.015 Pa and increasing

For a sound wave, the excess pressure $p(x,t)$ and displacement $s(x,t)$ are related such that pressure is 90° ($\pi/2$ radians) out of phase with displacement.
Using $p(x,t) = P_{max}\sin(\phi)$ and $s(x,t) = s_{max}\cos(\phi)$:
Given $p = 0.015$ Pa and $P_{max} = 0.03$ Pa:

$$\sin(\phi) = \frac{0.015}{0.03} = 0.5$$

For pressure to be **increasing**, the phase ϕ must be in the first quadrant (30° or $\pi/6$).

$$s = s_{max}\cos(30^\circ) = (1.16 \times 10^{-8})\left(\frac{\sqrt{3}}{2}\right)$$

$s \approx 1.00 \times 10^{-8}$ m

(b) Displacement when pressure is 0.015 Pa and decreasing

If the pressure is **decreasing** at $p = 0.5P_{max}$, the phase ϕ must be in the second quadrant (150° or $5\pi/6$).

$$s = s_{max}\cos(150^\circ) = (1.16 \times 10^{-8})\left(-\frac{\sqrt{3}}{2}\right)$$

$s \approx -1.00 \times 10^{-8}$ m

(c) Excess density or deficit

Excess density $\Delta\rho$ is directly proportional to excess pressure: $\Delta\rho = \frac{p}{v^2}$.
Since $p = +0.015$ Pa is positive in both cases (a) and (b), there is an **excess density** (compression).

$$\Delta\rho = \frac{0.015}{344^2}$$

$$\Delta\rho \approx 1.27 \times 10^{-7}\ \text{kg/m}^3$$

Problem 3
The sound wave of problem 1 is traveling down a hollow tube of cross-sectional
area 10cm^2 . Consider a small segment of that tube of length Δx=1cm.
(a) At an instant where the pressure in Δx is maximum, what is the net force acting
on the air in that segment of the tube?
(b) At an instant where the pressure in Δx is zero, what are the possible values of
the net force acting on that segment?
(c) For case (b), what are the possible values of the acceleration of the air molecules at that instant?
(d) I said the segment of the tube we are considering, of length Δx=1cm, is “small”. Why does it need to be small? Small compared to what? Explain clearly what goes wrong if it’s not “small”.

(a) Net force when pressure is maximum
When the pressure within the segment Δx is at its maximum value ($P_{max} = 0.03$ Pa), the pressure gradient at that specific point is zero ($\frac{\partial p}{\partial x} = 0$). Since the pressure is the same on both faces of the small segment:

$$F_{net} = (p_{left} - p_{right})A \approx 0$$

$F_{net} = 0$ N

(b) Net force when pressure is zero
When the pressure at the center of the segment is zero ($p = 0$), the pressure gradient is at its maximum magnitude. The gradient is $|\frac{\partial p}{\partial x}| = kP_{max}$.

The net force is $F_{net} = -\left(\frac{\partial p}{\partial x}\right)\Delta x \cdot A$.

Using $k = \frac{2\pi f}{v} = \frac{2\pi(1000)}{344} \approx 18.27 \text{ m}^{-1}$:

$$|F_{net}| = (kP_{max})\Delta x \cdot A = (18.27 \times 0.03)(0.01)(0.001)$$

$|F_{net}| \approx 5.48 \times 10^{-6}$ N (Direction depends on whether pressure is increasing or decreasing with x).

(c) Possible values of acceleration
Using Newton's Second Law, $a = \frac{F_{net}}{m}$, where $m = \rho A \Delta x$:

$$m = (1.20)(0.001)(0.01) = 1.2 \times 10^{-5} \text{ kg}$$

$$a = \pm \frac{5.48 \times 10^{-6}}{1.2 \times 10^{-5}}$$

$a \approx \pm 0.457 \text{ m/s}^2$

(d) Why Δx must be "small"

The segment must be small compared to the **wavelength** (λ) of the sound wave. For this wave:

$$\lambda = \frac{v}{f} = \frac{344}{1000} = 0.344 \text{ m} = 34.4 \text{ cm}$$

Our $\Delta x = 1$ cm is small relative to 34.4 cm.

What goes wrong: If Δx is not small (e.g., $\Delta x \approx \lambda/2$), the pressure would vary significantly across the segment. The linear approximation for the pressure difference, $\Delta p \approx \frac{\partial p}{\partial x} \Delta x$, would fail because the pressure gradient is not constant over the segment's length. One would have to integrate the pressure over the volume, and the net force could cancel out even if the local pressure gradient is high.

Problem 4
(a) What is the intensity of that wave, in dB?
(b) How much energy crosses a cross-section of the tube per unit time (time averaged)?

(a) Intensity of the wave (in dB)

First, calculate the intensity I in W/m^2 using the pressure amplitude P_{max}:

$$I = \frac{P_{max}^2}{2\rho v} = \frac{0.03^2}{2(1.20)(344)} \approx 1.09 \times 10^{-6} \text{ W/m}^2$$

Now, convert this to the decibel scale (β) using the reference intensity $I_0 = 10^{-12} \text{ W/m}^2$:

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right) = 10 \log_{10}\left(\frac{1.09 \times 10^{-6}}{10^{-12}}\right)$$

$\boldsymbol{\beta \approx 60.4}$ **dB**

(b) Energy crossing the cross-section per unit time

The time-averaged energy per unit time is the power P_{avg}, which is intensity multiplied by the cross-sectional area $A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$:

$$P_{avg} = I \cdot A = (1.09 \times 10^{-6} \text{ W/m}^2)(10 \times 10^{-4} \text{ m}^2)$$

$\boldsymbol{P_{avg} \approx 1.09 \times 10^{-9}}$ **J/s (or Watts)**

Problem 5
Assume you are talking at a volume of 60dB. Estimate how long you would have to talk continuously to burn off the energy in a single M&M candy, about 3.4 kcal.
(1kcal=4184 J). Assume the sound exits your mouth through an area of 5cm2, and
that your body is 100% efficient in converting food energy into sound energy

Time required to burn off one M&M

First, convert the intensity of the sound from decibels to $\mathrm{W/m^2}$:

$$60\ \mathrm{dB} = 10\log_{10}\left(\frac{I}{I_0}\right) \implies I = 10^{-6}\ \mathrm{W/m^2}$$

Calculate the total power P produced by talking, using the area $A = 5\ \mathrm{cm^2} = 5 \times 10^{-4}\ \mathrm{m^2}$:

$$P = I \cdot A = (10^{-6}\ \mathrm{W/m^2})(5 \times 10^{-4}\ \mathrm{m^2}) = 5 \times 10^{-10}\ \mathrm{W\ (J/s)}$$

Convert the energy in one M&M ($3.4\ \mathrm{kcal}$) into Joules:

$$E = 3.4\ \mathrm{kcal} \times 4184\ \mathrm{J/kcal} \approx 14,225.6\ \mathrm{J}$$

Calculate the time t in seconds:

$$t = \frac{E}{P} = \frac{14,225.6}{5 \times 10^{-10}} = 2.845 \times 10^{13}\ \mathrm{seconds}$$

Convert the time into years:

$$t \approx \frac{2.845 \times 10^{13}}{3.15 \times 10^{7}\ \mathrm{s/year}}$$

$$t \approx 903,000 \text{ years}$$