PHYSICS 210B : NONEQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #3 : FOKKER-PLANCK EQUATION

(1) Consider the following stochastic differential equation,

$$dx = -\beta x\,dt + \sqrt{2\beta(a^2 - x^2)}\,\eta(t)\,dt\ ,$$

where $x \in [-a, a]$ and $\langle \eta(t)\,\eta(t')\rangle = \delta(t - t')$.

(i) Find the corresponding Fokker-Planck equation.

(ii) Find the normalized steady state probability $\mathcal{P}(x)$.

(iii) Find and solve for the eigenfunctions $P_n(x)$ and $Q_n(x)$. *Hint: learn a bit about Chebyshev polynomials.*

(iv) Find an expression for $\big\langle x^3(t)\,x^3(0)\big\rangle$, assuming $x_0 \equiv x(0)$ is distributed according to $\mathcal{P}(x_0)$.

(2) A diffusing particle is confined to the interval $[0, L]$. The diffusion constant is D and the drift velocity is $v_{\rm D}$. The boundary at $x = 0$ is absorbing and that at $x = L$ is reflecting.

(a) Calculate the mean and mean square time for the particle to get absorbed at $x = 0$ if it starts at $t = 0$ from $x = L$. Examine in detail the cases $v_{\rm D} > 0$, $v_{\rm D} = 0$, and $v_{\rm D} < 0$.

(b) Compute the Laplace transform of the distribution of trapping times for the cases $v_{\rm D} > 0$, $v_{\rm D} = 0$, and $v_{\rm D} < 0$, and discuss the asymptotic behaviors of these distributions in the limits $t \to 0$ and $t \to \infty$.