Physics 2D Quiz 5 Solutions Winter 2025

Problem 1

$$\psi(x) = A(x^2 - dx)\, e^{-x/2a_0} \quad . \quad a_0 = \hbar^2/m_e k e^2$$

satisfies the Schrödinger eq. $-\frac{\hbar^2}{2m_e}\frac{d^2\psi}{dx^2} - \frac{ke^2}{x}\psi = E\psi$

we can ignore A, since it cancels

$$\frac{d\psi}{dx} = (2x-d)\,e^{-x/2a_0} - \frac{1}{2a_0}(x^2-dx)\,e^{-x/2a_0}$$

$$\boxed{\frac{d^2\psi}{dx^2} = \left[-\frac{1}{a_0}(2x-d) + 2 + \frac{1}{4a_0^2}(x^2-dx)\right]e^{-x/2a_0}}$$

Substituting in Schrod eq, the exponential cancels from all terms $\Rightarrow$

$$\boxed{\frac{\hbar^2}{2m_e a_0}(2x-d) - \frac{\hbar^2}{m_e} - \frac{\hbar^2}{8m_e a_0^2}(x^2-dx) - ke^2(x-d) = E(x^2-dx)}$$

There are only 2 terms with x^2, they have to be equal:

$$-\frac{\hbar^2}{8m_e a_0^2}x^2 = Ex^2 \Rightarrow E = -\frac{\hbar^2}{8m_e a_0^2} = -\frac{E_0}{4}$$

$$\Rightarrow \boxed{E = -3.40\,\text{eV}}$$

For extra credit: we can set all constant terms in the eq. to be equal

$$-\frac{\hbar^2}{2m_e a_0}d - \frac{\hbar^2}{m_e} + ke^2 d = 0. \quad \text{Since } ke^2 = \hbar^2/m_e a_0,$$

$$-\frac{\cancel{\hbar^2}}{2\cancel{m_e}a_0}d + \frac{\cancel{\hbar^2}}{\cancel{m_e}a_0}d = \frac{\cancel{\hbar^2}}{\cancel{m_e}} \Rightarrow -\frac{d}{2a_0} + \frac{d}{a_0} = 1 \Rightarrow \frac{d}{2a_0} = 1 \Rightarrow$$

$$\Rightarrow \boxed{d = 2a_0 = 0.1058\,\text{nm}}$$

Problem 2

electron has quantum #'s $n = 4$, $m_\ell = 1$

That means $\ell = 1$ or $\ell = 2$ or $\ell = 3$, since $-\ell \le m_\ell \le \ell$

The angle that $\vec{L}$ makes with the z axis is

$$\cos\theta = \frac{L_z}{|\vec{L}|} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}} = \begin{cases} \frac{1}{\sqrt{2}} & \text{for } \ell = 1 \\ 1/\sqrt{6} & \text{for } \ell = 2 \\ 1/\sqrt{12} & \text{for } \ell = 3 \end{cases}$$

The angles are $\theta = 45°$, $\theta = 65.9°$, or $\theta = 73.2°$

(b) The radial wave function has no nodes for $\ell = n - 1 = 3$

" " " " " one " " $\ell = n - 2 = 2$

" " " " " two " " $\ell = n - 3 = 1$

so by looking at the number of nodes in the graph of $R(r)$ we can tell whether is is $\ell = 1$ or $\ell = 2$ or $\ell = 3$.

Problem 3

$$R(r) = Cr^2 e^{-r/a_0}$$

$$P(r) = r^2 R^2(r) = C^2 r^6 e^{-2r/a_0}$$

most probable r:

$$P'(r) = 0 = 6r^5 - \frac{2r^6}{a_0} \Rightarrow \boxed{r = 3a_0} \quad (a)$$

average r:

$$\langle r \rangle = \frac{\int_0^\infty dr\, r\, P(r)}{\int_0^\infty dr\, P(r)} = \frac{\int_0^\infty dr\, r^7 e^{-2r/a_0}}{\int_0^\infty dr\, r^6 e^{-2r/a_0}} =$$

$$= \frac{7!}{2^7} \frac{a_0^7}{6!} \cdot \frac{2^6}{a_0^6} = \frac{7}{2} a_0 = \boxed{3.5 a_0} \quad (b)$$

(c) In Bohr atom, $r = \frac{a_0}{Z} n^2$

In Schrodinger atom, $R(r) \sim e^{-Zr/na_0}$

Here, since $R(r) \sim e^{-r/a_0} \Rightarrow n = Z$

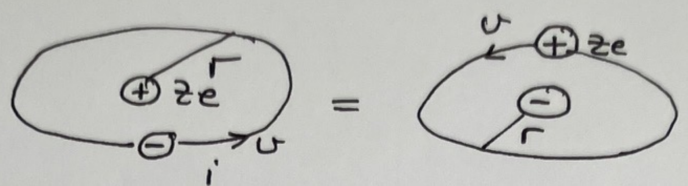
If the most probable radius agrees with the value in the Bohr model

$\Rightarrow \quad \frac{a_0}{Z} n^2 = 3a_0$, and $n = Z \Rightarrow a_0 Z = 3a_0$

$$\Rightarrow \boxed{Z = 3} \quad (c)$$

Problem 4

$$B = \frac{\mu_0 i}{2r}$$



Current from charge Ze moving with speed v in circle of radius r:

$$i = \frac{Zev}{2\pi r} \quad . \quad \text{So} \quad B = \frac{\mu_0 Z e v}{4\pi r^2}$$

The angular momentum in the z direction is

$$L_z = m_\ell \hbar = m_e v r \Rightarrow v = \frac{m_\ell \hbar}{m_e r} \Rightarrow$$

$$\Rightarrow B = \frac{\mu_0 Z e \hbar}{4\pi m_e r^3} m_\ell$$

In a hydrogen-like ion, the radius according to Bohr's model is

$$r_n = \frac{a_0}{Z} n^2 . \quad \text{So}$$

$$\boxed{B = \frac{\mu_0 e \cancel{Z} \hbar}{4\pi m_e a_0^3} \frac{Z^4}{n^6} m_\ell}$$

(a) if $Z = 1 \to Z = 2$, $B \to 2^4 B = 16 \times 0.39\,T = \boxed{6.24\,T}$

(b) If $n = 2$, $m_\ell = 1 \to n = 3$, $m_\ell = 2$,

$$\frac{m_\ell}{n^6} = \frac{1}{2^6} \to \frac{2}{3^6} \Rightarrow B \to B \cdot \frac{2}{3^6} \times 2^6 = 0.176\,B = \boxed{0.068\,T}$$

Problem 5

The K_α line has wavelength λ that satisfies

$$\frac{hc}{\lambda} = 13.6\,eV\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

So $$\lambda_{Ni}^{K\alpha} = \lambda_{Fe}^{K\alpha} \times \frac{(Z_{Fe}-1)^2}{(Z_{Ni}-1)^2} = \frac{25^2}{27^2} \times 0.1945\,nm$$

$$\Rightarrow \boxed{\lambda_{Ni}^{K\alpha} = 0.1668\,nm} \quad (a)$$

(b) The K_β line has wavelength λ that satisfies

$$\frac{hc}{\lambda} = 13.6\,eV\left(\frac{1}{1^2} - \frac{1}{3^2}\right)(Z-1)^2$$

So $$\lambda^{K\beta} = \lambda^{K\alpha} \times \frac{(1-\frac{1}{4})}{(1-\frac{1}{9})} = \frac{3\cdot 9}{4\cdot 8}\lambda^{K\alpha} = \frac{27}{32}\lambda^{K\alpha} = \frac{27}{32} \times 0.1945\,nm$$

$$\Rightarrow \boxed{\lambda_{Fe}^{K\beta} = 0.1641\,nm} \quad (b)$$