PHYSICS 2D QUIZ 5 WINTER QUARTER 2025
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Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6\times 10^{-19} J$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037\times 10^{-8} W/m^2\cdot K^4$; Wien's law: $\lambda_m T = 2.8978\times 10^{-3} m\cdot K$

$I(T) = \int_0^\infty I(\lambda,T)\,d\lambda$; $I = (c/4)u$; $u(\lambda,T) = N(\lambda)E_{av}(\lambda,T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\,dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\,dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^{\infty} N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^{\infty} E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240 eV\cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d\sin\theta = n\lambda$; single-slit diffraction minima: $a\sin\theta = n\lambda$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$ $\hbar c = 197.3$ eV nm

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x \Delta k \sim 1$ $\Delta t \Delta\omega \sim 1$; $\Delta x \Delta p \sim \hbar$ $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$; wave packets: $\psi(x,t) = \int a(k)\ e^{i(kx-\omega(k)t)}$

Schrodinger equation: $-\dfrac{\hbar^2}{2\mathrm{m}}\dfrac{\partial^2\Psi}{\partial x^2} + \mathrm{U(x)}\Psi(\mathrm{x,t}) = \mathrm{i}\hbar\ \dfrac{\partial\Psi}{\partial t}$; $\Psi(\mathrm{x,t}) = \psi(\mathrm{x})\mathrm{e}^{-\mathrm{i}\frac{\mathrm{E}}{\hbar}t}$

Time – independent Schrodinger equation: $-\dfrac{\hbar^2}{2\mathrm{m}}\dfrac{\partial^2\psi}{\partial x^2} + \mathrm{U(x)}\psi(\mathrm{x}) = \mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $< f(x) > = \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\dfrac{2}{L}}\sin(\dfrac{n\pi x}{L})$; $E_n = \dfrac{\pi^2\hbar^2 n^2}{2mL^2}$; $\dfrac{\hbar^2}{2m_e} = 0.0381 eVnm^2$ (electron)

2D square well: $\Psi_{n_1n_2}(\mathrm{x,y}) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $\mathrm{E}_{n_1n_2} = \dfrac{\pi^2\hbar^2}{2m}(\dfrac{n_1^2}{L_1^2} + \dfrac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w) = \sqrt{\dfrac{2}{L_i}}\sin(\dfrac{n\pi w}{L_i})$

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\dfrac{1}{2})\hbar\omega$; $E = \dfrac{p^2}{2m} + \dfrac{1}{2}m\omega^2 x^2 = \dfrac{1}{2}m\omega^2 A^2$

Step potential: reflection coef : $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\dfrac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim \mathrm{e}^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x1}^{x2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\dfrac{2m[U(x) - E]}{\hbar^2}}$

Rutherford scattering: $U = \dfrac{2Ze^2}{4\pi\varepsilon_0 r}$; $e^2/(4\pi\varepsilon_0) = 1.44 eV\cdot nm$

$b = \dfrac{Z}{K_\alpha}\dfrac{e^2}{4\pi\varepsilon_0}\cot\dfrac{1}{2}\theta$; $f_{>\theta} = nt\pi b^2$; $N(\theta) = \mathrm{constant} \times (\dfrac{Z}{K_\alpha})^2 \times \dfrac{1}{\sin^4(\theta/2)}$

Line spectra: $\dfrac{1}{\lambda} = R(\dfrac{1}{n_0^2} - \dfrac{1}{n^2})$; $R = \dfrac{1}{91.13nm}$; $hcR = 13.6eV$

Bohr atom: $F = \dfrac{ke^2Z}{r^2} = m_e\dfrac{v^2}{r}$; $U = -\dfrac{ke^2Z}{r}$; $E = K + U = -\dfrac{ke^2Z}{2r}$; $a_0 = 0.0529nm$

$r_n = (a_0/Z)n^2$; $E_n = -E_0 Z^2/n^2$; $E_0 = \dfrac{ke^2}{2a_0}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2}$; $L = m_e vr = n\hbar$; $E_0 = -13.6eV$

1-dim atom: $U(x) = -ke^2/x$; ground state wavefunction $\psi(\mathrm{x}) = \mathrm{Axe}^{-x/a_0}$

Spherically symmetric potential: $\Psi_{\mathrm{n},\ell,\mathrm{m}_\ell}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell,\mathrm{m}_\ell}(\theta,\phi)$; $Y_{\ell,\mathrm{m}_\ell}(\theta,\phi) = P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

quantum numbers: $n = 1,2,3,...$; $0 \le \ell \le n-1$; $-\ell \le m_\ell \le \ell$

Angular momentum: $\vec{\mathrm{L}} = \vec{r}\times\vec{p}$; $|L^2| = \ell(\ell+1)\hbar^2$; $\mathrm{L_z} = m_\ell\hbar$

Radial probability density: $P(r) = r^2 \,|R_{n\ell}(r)|^2$; Energy: $E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$

Radial wave function: $R_{nl}(r)$ = (n-th degree polynomial in r) × e^{-Zr/na_0}

Ground state of hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$; $\int_0^\infty dr\, r^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79\times10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|\vec{S}| = \sqrt{s(s+1)}\hbar$; $S_z = m_s\hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e}g\vec{S}$; $g = 2$

Orbital+spin mag moment: $\vec{\mu} = \frac{-e}{2m_e}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu}\cdot\vec{B}$

subshells: 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d

$2(2\ell+1)$ electrons per subshell. Screened electron: $E_n = (-13.6eV)Z_{eff}^2/n^2$

Problem 1 (6 points)
The wavefunction for an electron in an excited state of a one-dimensional atom with potential energy $U(x) = -ke^2/x$ is $\psi(x) = A(x^2 - qx)e^{-x/2a_0}$
where $a_0 = \hbar^2/m_e ke^2$ is the Bohr radius, and A and q are constants.
Find the energy of this electron, in eV. Justify your answer.
Hints: (i) Use the Schrodinger equation; (ii) Consider the terms that dominate for large x values; (iii) You don't need to know the value of q.
For extra credit: do find the value of q, in nm. Justify your answer.

Problem 2 (6 points)
An electron is in a state of a hydrogen-like ion with quantum numbers n=4 and $m_\ell = 1$.
(a) Find all the possible values for the angle that its angular momentum vector $\vec{L}$ could be making with the z axis. Give your answers in degrees. Justify your answers.
(b) Suppose you also have a graph showing the radial wave function of this electron as a function of r. Explain how you could tell by looking at this graph what is the angular momentum quantum number ℓ for this electron.

Problem 3 (6 points)
The radial wave function of an electron in a hydrogen-like ion is $R(r) = Cr^2e^{-r/a_0}$.
(a) Find the most probable radius r for this electron, in terms of a_0.
(b) Find the average radius <r> for this electron, in terms of a_0.
(c) If the most probable radius is the same as in the Bohr atom, what is the value of Z?

Problem 4 (6 points)
For an electron in the hydrogen atom (Z=1) in the state with quantum numbers

$n=2,\ \ell=1,\ m_\ell=1$, the magnetic field acting on the electron spin due to its orbital motion around the nucleus (no external magnetic field) has magnitude B=0.39T.
(a) What would be the magnetic field if the electron was in this state in a hydrogen-like ion with Z=2 instead? Justify your answer.
(b) What would be the magnetic field for this electron in hydrogen (Z=1) if it was in the state with quantum numbers $n=3,\ \ell=2,\ m_\ell=2$ instead? Justify your answer.
Hints: (i) Use that the magnetic field at the center of a loop of radius r carrying current i is $B=\mu_0 i/(2r)$; (ii) Use Bohr atom relations for the radius, Schrodinger atom relation for the z component of angular momentum. (iii) As you know, the current created by a charge q moving at speed v in a circle of radius r is $i=qv/(2\pi r)$.

Problem 5 (6 points)
The K_α line in the X-ray spectrum of iron (Z=26) has wavelength $\lambda=0.1945nm$.
(a) What do you expect will be the wavelength of the K_α line of nickel (Z=28)?
(b) What do you expect will be the wavelength of the K_β line of iron?