Physics 2D — Quiz 4 Solutions — Winter 2025

Problem 1

The classical amplitude is given by $x_0 = A$, satisfying

$$E = \frac{1}{2} m_e \omega^2 A^2$$

In the ground state, $E = E_0 = \frac{\hbar\omega}{2}$, so, with $A =$

$$\frac{\hbar\omega}{2} = \frac{1}{2} m_e \omega^2 A^2 \Rightarrow \omega = \frac{\hbar}{m_e A^2} \Rightarrow \hbar\omega = \frac{\hbar^2}{m_e A^2} = \frac{2 \times 0.0381\, eV\, nm^2}{0.2^2\, nm^2}$$

$$\Rightarrow \hbar\omega = 1.905\, eV$$

Harmonic oscillator only absorbs photons of frequency ω, wavelength λ, with

$$\frac{hc}{\lambda} = \hbar\omega \Rightarrow \lambda = \frac{hc}{\hbar\omega} = \frac{1240\, eV\, nm}{1.905\, eV} = 650.9\, nm$$

So only 1 absorption line is seen, $\boxed{\lambda = 650.9\, nm}$

(b) Before absorbing radiation, electron in ground state with

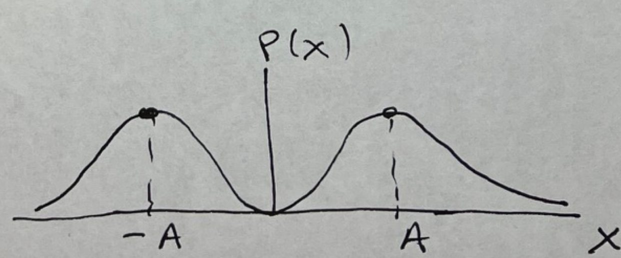
$$\psi_0(x) = C_0 e^{-x^2/2A^2} \Rightarrow \text{most likely to be found at } \boxed{x = 0}$$

After absorbing a photon, electron is in state

$$\psi_1(x) = C_1 x e^{-x^2/2A^2}.$$ Probability is

$$P(x) = |\psi_1(x)|^2 = C_1^2 x^2 e^{-x^2/A^2}$$

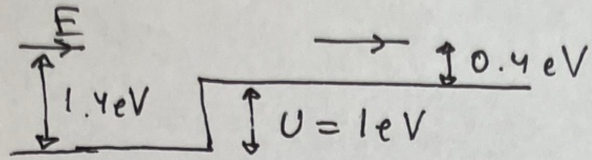
find maximum:



$$P'(x) = 0 = 2x - \frac{2x^3}{A^2} = 0$$

$$\Rightarrow x^2 = A^2 \Rightarrow \boxed{x = \pm A}$$

Problem 2



incident energy is $E = 1.4\,eV$

Reflection coef $R = \dfrac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$

$$k_1 = \sqrt{\frac{2m_e E}{\hbar^2}} = \sqrt{\frac{1.4\,eV}{0.0381\,eV\,nm^2}} = 6.062\,nm^{-1}$$

$$k_2 = \sqrt{\frac{2m_e (E-U)}{\hbar^2}} = \sqrt{\frac{0.4\,eV}{0.0381\,eV\,nm^2}} = 3.240\,nm^{-1}$$

$$R = \left(\frac{6.062 - 3.24}{6.062 + 3.24}\right)^2 = 0.092$$

transmission coef $= T = 1 - R = 0.908$

If 1000 electrons were transmitted, the number of incident electrons was N such that $N \cdot T = 1000 \Rightarrow$

$$N = \frac{1000}{0.908} = \boxed{1{,}101 \text{ electrons}} \text{ were incident}$$

Problem 3

According to the Rutherford formula, # of α particles scattered at angle θ is proportional to $\frac{1}{\sin^4 \theta/2}$

$$\Rightarrow \quad \frac{N(\theta=90^\circ)}{N(\theta=180^\circ)} = \frac{\sin^4(90^\circ)}{\sin^4(45^\circ)} = \frac{1}{(\sqrt{2}/2)^4} = 4$$

so according to the problem, this is what is observed for kinetic energy of α particle $K_\alpha = 4\,MeV$.

The distance of closest approach to the center of the nucleus for impact parameter $b=0$ is r, satisfying

$$K_\alpha = \frac{2Ze^2}{4\pi\varepsilon_0 r} \Rightarrow r = \frac{2Ze^2}{4\pi\varepsilon_0 K_\alpha} = \frac{2\times 47\times 1.44\,eV.nm}{K_\alpha}$$

$$\Rightarrow r = \frac{135.36\,eV.nm}{K_\alpha}$$

. For the kinetic energies given in the problem

K_α	r	Is Rutherford law satisfied? it is if α particles don't penetrate the nucleus
3.5 MeV	3.86×10^{-14} m	yes
4 MeV	3.38×10^{-14} m	yes
4.5 MeV	3.01×10^{-14} m	?
5 MeV	2.71×10^{-14} m	no

(a) From the fact that Rutherford law is satisfied for $K_\alpha = 4\,MeV$ and not satisfied for $K_\alpha = 5\,MeV$ we deduce that the radius of the nucleus of Ag, r_{Ag}, satisfies: $\boxed{2.71\cdot10^{-14}\,m < r_{Ag} < 3.38\times10^{-14}\,m}$

(b) For $K_\alpha = 3.5\,MeV$, the α particles stay outside nucleus $\Rightarrow \frac{N(90^\circ)}{N(180^\circ)} = 4$

For $K_\alpha = 4.5\,MeV$, we don't know whether α-particles penetrate the nucleus, so all we know is $\boxed{3.8 < \frac{N(90^\circ)}{N(180^\circ)} \leq 4}$

Problem 4

$E = -13.6\,eV = -E_o$, $r = 0.2116\,nm = 4a_o$

We know that:

$$E_n = -\frac{E_o Z^2}{n^2} = -E_o \Rightarrow Z = n$$

and that $r_n = \frac{a_o}{Z} n^2 = 4a_o \Rightarrow n^2 = 4Z = 4n \Rightarrow$

$$\Rightarrow \boxed{n = 4,\ Z = 4}$$

(a) angular momentum is $\boxed{L_n = n\hbar = 4\hbar}$

(b) The shortest wavelength is for the transition $n = 4$ to $n = 1$

$$\frac{hc}{\lambda} = E_4 - E_1 = E_o Z^2\left(1 - \frac{1}{4^2}\right) = 0.9375\,E_o Z^2$$

$$\Rightarrow \lambda = \frac{hc}{0.9375\,E_o Z^2} = \frac{1240\,eV\,nm}{0.9375 \times 13.6\,eV \times 16}$$

$$\Rightarrow \boxed{\lambda = 6.078\,nm}$$

Problem 5

$F = -kr \Rightarrow$

$kr = \frac{m_e v^2}{r} \Rightarrow \boxed{kr^2 = m_e v^2}$

$\Rightarrow \boxed{K = \frac{1}{2} m_e v^2 = \frac{1}{2} k r^2}$ (a)

(b) Assuming $L = n\hbar$ as in the Bohr atom,

$L = m_e v r = n\hbar \Rightarrow L^2 = m_e^2 v^2 r^2 = n^2 \hbar^2 \Rightarrow$

$\Rightarrow m_e k r^2 r^2 = n^2 \hbar^2 \Rightarrow r_n^4 = \frac{\hbar^2}{m_e k} n^2 \Rightarrow \boxed{r_n = \left(\frac{\hbar^2}{m_e k}\right)^{1/4} n^{1/2}}$

smallest orbit is for $n = 1$

$r_1^4 = \frac{\hbar^2}{m_e k} \Rightarrow \boxed{r_1 = \left(\frac{\hbar^2}{m_e k}\right)^{1/4}}$

For $k = 1 \frac{eV}{nm^2}$, $\boxed{r_1 = \left(\frac{2 \times 0.0381\, eV\, nm^2 . nm^2}{\cancel{} . 1 eV}\right)^{1/4} = 0.525\, nm}$