Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6 \times 10^{-19} J$

Photoelectric effect: $eV_s = K_{max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} W/m^2 \cdot K^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} m \cdot K$

$I(T) = \int_0^\infty I(\lambda, T)\, d\lambda$; $I = (c/4)u$; $u(\lambda, T) = N(\lambda) E_{av}(\lambda, T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\, dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\, dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240 eV \cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d\sin\theta = n\lambda$; single-slit diffraction minima: $a\sin\theta = n\lambda$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$ $\hbar c = 197.3$ eV nm

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x \Delta k \sim 1$ $\Delta t \Delta\omega \sim 1$; $\Delta x \Delta p \sim \hbar$ $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{<A^2> - <A>^2}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; wave packets: $\psi(x,t) = \int a(k)\ e^{i(kx-\omega(k)t)}$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $< f(x) > = \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 0.0381 eVnm^2$ (electron)

2D square well: $\Psi_{n_1 n_2}(x,y) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $E_{n_1 n_2} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}}\sin(\frac{n\pi w}{L_i})$

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$

Step potential: reflection coef : $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Rutherford scattering: $U = \frac{2Ze^2}{4\pi\varepsilon_0 r}$; $e^2/(4\pi\varepsilon_0) = 1.44 eV\cdot nm$

$b = \frac{Z}{K_\alpha}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$; $f_{>\theta} = nt\pi b^2$; $N(\theta) = \text{constant}\times(\frac{Z}{K_\alpha})^2 \times \frac{1}{\sin^4(\theta/2)}$

Line spectra: $\frac{1}{\lambda} = R(\frac{1}{n_0^2} - \frac{1}{n^2})$; $R = \frac{1}{91.13nm}$; $hcR = 13.6eV$

Bohr atom: $F = \frac{ke^2 Z}{r^2} = m_e\frac{v^2}{r}$; $U = -\frac{ke^2 Z}{r}$; $E = K + U = -\frac{ke^2 Z}{2r}$; $a_0 = 0.0529nm$

$r_n = (a_0/Z)n^2$; $E_n = -E_0 Z^2/n^2$; $E_0 = \frac{ke^2}{2a_0}$; $a_0 = \frac{\hbar^2}{m_e ke^2}$; $L = m_e vr = n\hbar$; $E_0 = 13.6eV$

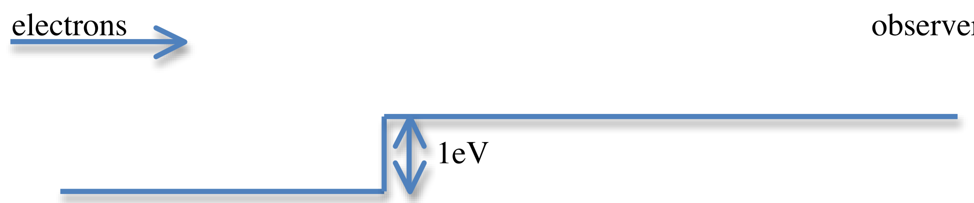
1-dim atom: $U(x) = -\frac{k}{x}$; $\psi(x) = Axe^{-bx}$

PHYSICS 2D QUIZ 4 WINTER QUARTER 2025
PROF. HIRSCH

Problem 1 (6 points)

An electron is in the ground state of a one-dimensional harmonic oscillator potential. Its classical turning points are at $x_0 = \pm 0.2nm$ (i.e. its classical amplitude of oscillation is 0.2 nm).
(a) If light in the visible wavelength range (400 nm to 800 nm) is incident on this system, absorption lines will be seen at which wavelengths?
(b) At which positions x is this electron most likely to be found (i) before absorbing radiation and (ii) after absorbing radiation? Justify your answers.

Problem 2 (6 points)



The picture above shows a potential step of height 1eV. The observers on the right see 1000 electrons flying by them, with kinetic enerqy 0.4 eV. How many electrons were incident from the left, and what was their kinetic energy?

Problem 3 (6 points)
In a Rutherford scattering experiment with a steady flow of α particles of kinetic energy 4MeV incident on a silver foil (Z=47 for Ag) it is found that there are approximately 4 times more α particles scattered at angle 90^0 than at 180^o. When the kinetic energy of the α particles increases to 5MeV, that ratio drops from 4 to 3.8.
(a) What can you deduce about the radius of the Ag nucleus from this information?
(b) What can you deduce about what the ratio of α particles scattered at angle 90^0 versus at 180^o will be for α particles of kinetic energy (i) 3.5 MeV and (ii) 4.5MeV?

Problem 4 (6 points)
An electron in a hydrogen-like ion has energy -13.6eV, and the radius of its orbit is 0.2116 nm.
(a) What is its angular momentum?
(b) What is the wavelength of the shortest wavelength photon that this ion can emit?

Problem 5 (6 points)
An electron moves in two dimensions subject to a central force $F = -\gamma r$, where γ is a constant and r the distance to the origin. Assume it moves in a circular orbit of radius r, with the central force providing the centripetal acceleration.
(a) Give an expression for the kinetic energy of this electron in terms of γ and r.
(b) If you assume that its angular momentum is quantized as it is in the Bohr atom, find an expression for the radius of the smallest possible orbit and its numerical value if $\gamma = 1eV/nm^2$. Give your answer in nm.