

Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$; $v'_y = \frac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \rightarrow -u$, primed \leftrightarrow unprimed; Doppler: $f' = f \sqrt{\frac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$
 $E = \sqrt{p^2 c^2 + m^2 c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 \text{ MeV}/c^2$; Proton: $m_p = 938.26 \text{ MeV}/c^2$; Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic unit: $1u = 931.5 \text{ MeV}/c^2$; electron volt: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$

$I(T) = \int_0^\infty I(\lambda, T) d\lambda$; $I = (c/4)u$; $u(\lambda, T) = N(\lambda)E_{av}(\lambda, T)$; $N(\lambda) = \frac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E) dE$; $E_{av} = \frac{1}{N} \int_0^\infty EN(E) dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \frac{1}{N} \sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1,240 \text{ eV} \cdot \text{nm}$; $\lambda_m T = hc/4.96k$; $\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$

Boltzmann constant: $k = (1/11,604) \text{ eV}/\text{K}$; $1\text{\AA} = 1\text{A} = 0.1 \text{ nm}$

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \frac{h}{m_e c} = 0.00243 \text{ nm}$

double-slit interference maxima: $d \sin\theta = n\lambda$; single-slit diffraction minima: $a \sin\theta = n\lambda$

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $\hbar c = 197.3 \text{ eV nm}$

matter: $E = \frac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2 c^2 + m^2 c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x \Delta k \sim 1$; $\Delta t \Delta \omega \sim 1$; $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; wave packets: $\psi(x,t) = \int a(k) e^{i(kx - \omega(k)t)}$

Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time - independent Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \psi^* \psi = 1$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx \quad ; \quad \langle f(x) \rangle = \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$$

$$\infty \text{ square well: } \psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad ; \quad E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2} \quad ; \quad \frac{\hbar^2}{2m_e} = 0.0381 \text{ eV nm}^2 \text{ (electron)}$$

$$2\text{D square well: } \Psi_{n_1 n_2}(x,y) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y) \quad ; \quad E_{n_1 n_2} = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} \right) \quad ; \quad \Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}} \sin\left(\frac{n\pi w}{L_i}\right)$$

For extra credit (6 points)

An electron's wavefunction $\psi(x,t)$ is described by the wavepacket

$$\psi(x,t) = \int_{-\infty}^{\infty} dk a(k) e^{i(kx - \omega(k)t)}$$

where

$$a(k) = C e^{-x_0^2(k-k_0)^2 - ikx_0}$$

and $x_0 = 6 \text{ nm}$, $k_0 = 0.3 \text{ nm}^{-1}$, and C is a constant.

- Find $\psi(x) = \psi(x,t=0)$, explain how you found it.
- Normalize $\psi(x)$
- Calculate $\langle x \rangle$, give your answer in nm.
- Calculate Δx , give your answer in nm.

$$\text{Use that } \int_{-\infty}^{\infty} dx e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}} \text{ and that } \int_{-\infty}^{\infty} dx x^2 e^{-\lambda x^2} = -\frac{\partial}{\partial \lambda} \int_{-\infty}^{\infty} dx e^{-\lambda x^2}$$

$$\text{and that } \sqrt{x^2 - \langle x \rangle^2} = \sqrt{\langle (x - \langle x \rangle)^2 \rangle}$$