Physics 2D   **Quiz 3 solutions**   Winter 2025

Problem 1

$\lambda = \frac{h}{p}$. $v = 0.6c \Rightarrow$ proton is relativistic.

$v = \frac{3}{5}c, \quad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{5}{4}$

$$p = \gamma m_p v = \frac{5}{4} \cdot \frac{3}{5} c \cdot m_p = \frac{3}{4} m_p c$$

$$\lambda = \frac{h}{\frac{3}{4} m_p c} = \frac{4}{3} \frac{hc}{m_p c^2} = \frac{4}{3} \frac{1240 \text{ eV nm}}{938.26 \cdot 10^6 \text{ eV}} = 1.76 \cdot 10^{-6} \text{ nm}$$

$$\boxed{\lambda = 1.76 \times 10^{-6} \text{ nm}}$$

(b) Total energy $E = \gamma m_p c^2 = \frac{5}{4} m_p c^2$

Kinetic energy $K = (\gamma - 1) m_p c^2 = \frac{1}{4} m_p c^2 = \frac{938.26}{4}$ MeV

$\Rightarrow K = 234.6$ MeV $\Rightarrow$ potential difference is

$$\boxed{\Delta V = 234.6 \times 10^6 \text{ V}}$$

Problem 2

$$\psi(x) = e^{ix/5x_0} e^{-x^2/x_0^2} \qquad x_0 = 5\,nm$$

The first factor is $e^{ik_0x} \Rightarrow k_0 = \frac{1}{5x_0} = \frac{1}{25}\,nm^{-1}$

$$p = \hbar k_0 = \frac{\hbar}{5x_0} = \frac{\hbar c}{5x_0 c} = \frac{197.3\,eV.nm}{25\,nm \cdot c} = 7.9\,eV/c$$

$\boxed{p = 7.9\,eV/c}$ (a)

(b) The uncertainty in the position is of order $\Delta x \sim x_0$

and $\Delta x \Delta p \sim \hbar \Rightarrow \Delta p \sim \frac{\hbar}{\Delta x} = \frac{\hbar}{x_0} = \frac{197.3\,eV\,nm}{5\,nm \cdot c}$

$\Rightarrow \boxed{\Delta p \sim 39.5\,eV/c}$

Or more precisely: for Gaussian, $\psi(x) \sim e^{-x^2/4\Delta x^2} \Rightarrow$

$\Rightarrow$ here $\Delta x = x_0/2$

and $\Delta x \Delta p = \frac{\hbar}{2} \Rightarrow \frac{x_0}{2}\Delta p = \frac{\hbar}{2} \Rightarrow \Delta p = \frac{\hbar}{x_0}$ same as above

Problem 3

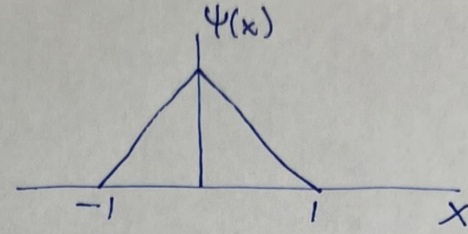
$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2 = \frac{0.0381\ eV\ nm^2}{0.04\ nm^2} \pi^2 n^2 = 9.40\ eV \cdot n^2$$

(a) $E_1 = 9.40\ eV$

(b) $E_2 = 4E_1 > 10\ eV \Rightarrow$ only one state with energy $< 10\ eV$.

Problem 4

$\psi(x) = C(1-|x|)$



$$\frac{1}{2} = \int_0^1 dx\, |\psi(x)|^2 = C^2 \int_0^1 dx\,(1-x)^2 = C^2 \int_0^1 dx\,(1-2x+x^2) =$$

$$= C^2\left[1-1+\frac{1}{3}\right] = \frac{C^2}{3} = \frac{1}{2} \Rightarrow C^2 = \frac{3}{2} \Rightarrow \boxed{C = \sqrt{\frac{3}{2}}}$$

(b) $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

by symmetry, $\langle x\rangle = 0$

$$\langle x^2\rangle = 2\int_0^1 dx\, x^2 |\psi(x)|^2 = 2 \cdot \frac{3}{2} \cdot \int_0^1 dx\, x^2(1-2x+x^2) =$$

$$= 3\left[\frac{1}{3} - \frac{\cancel{2}^{1}}{\cancel{4}_{2}} + \frac{1}{5}\right] = 3\,\frac{10-15+6}{30} = \frac{1}{10}$$

$$\Rightarrow \boxed{\Delta x = \sqrt{\frac{1}{10}} = 0.316}$$

Problem 5

$$\psi_{n_1,n_2}(x,y) = \frac{2}{L}\sin\left(\frac{n_1\pi}{L}x\right)\sin\left(\frac{n_2\pi}{L}y\right)$$

For the ground state

$$\psi_0(x,y) = \psi_{11}(x,y) = \frac{2}{L}\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right)$$

Probability that electron is in small area A around (x,y)

$$P_A(x,y) = |\psi_0(x,y)|^2 \cdot A = \frac{4}{L^2}A \sin^2\frac{\pi}{L}x \sin^2\frac{\pi}{L}y$$

If $P_A(x,y) = A/L^2 \Rightarrow 4\sin^2\frac{\pi}{L}x \sin^2\frac{\pi}{L}y = 1$

$$\Rightarrow \boxed{\sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}y\right) = \frac{1}{2}}$$

5 points that satisfy that:

1) $x = y = \frac{L}{4}$, since $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

2) $x = y = \frac{3L}{4}$, since $\sin\left(\frac{3}{4}\pi\right) = \frac{1}{\sqrt{2}}$

3) $x = \frac{L}{4}$, $y = \frac{3L}{4}$

4) $x = \frac{3L}{4}$, $y = \frac{L}{4}$

5) $x = \frac{L}{6}$, $y = \frac{L}{2}$, since $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $\sin\frac{\pi}{2} = 1$