PHYSICS 2D QUIZ 3 WINTER QUARTER 2025
PROF. HIRSCH

<u>Justify all your answers to all problems. Write clearly.</u>

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \, m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6 \times 10^{-19} J$

Photoelectric effect: $eV_s = K_{max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} W/m^2 \cdot K^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} m \cdot K$

$I(T) = \int_0^\infty I(\lambda, T)\, d\lambda$; $I = (c/4)u$; $u(\lambda, T) = N(\lambda) E_{av}(\lambda, T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\, dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\, dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^{\infty} N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^{\infty} E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240 eV \cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d \sin\theta = n\lambda$; single-slit diffraction minima: $a \sin\theta = n\lambda$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$ $\hbar c = 197.3$ eV nm

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x \Delta k \sim 1$ $\Delta t \Delta \omega \sim 1$; $\Delta x \Delta p \sim \hbar$ $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; wave packets: $\psi(x,t) = \int a(k)\ e^{i(kx-\omega(k)t)}$

Schrodinger equation: $-\frac{\hbar^2}{2\text{m}}\frac{\partial^2\Psi}{\partial x^2} + \text{U(x)}\Psi(\text{x,t}) = \text{i}\hbar\frac{\partial\,\Psi}{\partial\,t}$; $\Psi(\text{x,t}) = \psi(\text{x})\text{e}^{-\text{i}\frac{\text{E}}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2\text{m}}\frac{\partial^2\psi}{\partial x^2} + \text{U(x)}\psi(\text{x}) = \text{E}\psi(\text{x})$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $<f(x)> = \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 0.0381 eVnm^2$ (electron)

2D square well: $\Psi_{n_1 n_2}(\text{x,y}) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $\text{E}_{n_1 n_2} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}}\sin(\frac{n\pi w}{L_i})$

<u>Problem 1</u> (6 points)
A proton is moving with speed v=0.6c.
(a) What is its de Broglie wavelength, in nm?
(b) Through what potential difference (in V) must the proton be accelerated to have that de Broglie wavelength?

<u>Problem 2</u> (6 points)
An electron's wavefunction $\psi(x,t)$ is described by a Gaussian wavepacket (reminder: a Gaussian wave packet is a superposition of plane waves with amplitudes $a(k) = e^{-\alpha^2(k-k_0)^2}$). At time t=0 the wavefunction of this electron is

$\psi(x,0) = e^{ix/(5x_0)}e^{-x^2/x_0^2}$ with $x_0 = 5nm$

(a) Estimate the momentum of this electron in units eV/c
(b) Estimate the uncertainty in the momentum of this electron in units eV/c

<u>Problem 3</u> (6 points)
An electron is in an infinite potential well of width 0.2 nm.
(a) Find the energy of this electron if it is in the ground state, in eV.
(b) How many states are there in this well for the electron with energy smaller than 10eV?

<u>Problem 4</u> (6 points)
A particle is described by the wavefunction

$\psi(x) = C|1-x|$ for $-1 \le x \le 1$

$\psi(x) = 0$ for $|x| > 1$

(a) Find C so that the wave function is normalized
(b) Find the uncertainty in the position of this particle Δx.
Hint: use that the wavefunction is even to simplify calculations.

Problem 5 (6 points)
Consider a particle in the two-dimensional potential well defined by the potential
$U(x,y) = 0$ for $0 \le x \le L$ and $0 \le y \le L$

$U(x,y) = \infty$ otherwise
If this was a classical particle, the probability to find it within a rectangular area A would be $P = A/L^2$.
For a quantum particle in the ground state of this potential find 5 different points (x,y) for which the probability to find the particle within a small rectangular area A ("small" meaning $A \ll L^2$) centered at those points equals the classical probability $P = A/L^2$.