Physics 2D | Quiz 2 Solutions | Winter 2025

Problem 1

$v_1 = 0.8c = \frac{4}{5}c \Rightarrow \gamma = \frac{1}{\sqrt{1 - v_1^2/c^2}} = \frac{5}{3}$. $m_1 = \frac{3}{20} M = 0.15 M$

Energy conservation:

$$Mc^2 = m_1 \gamma(v_1) c^2 + m_2 \gamma(v_2) c^2 \Rightarrow$$

$$M = \frac{\cancel{3}}{20} M \cdot \frac{5}{\cancel{3}} + m_2 \gamma(v_2) \Rightarrow \boxed{m_2 \gamma(v_2) = \frac{3}{4} M}$$

Momentum conservation:

$$0 = m_1 v_1 \gamma(v_1) - m_2 v_2 \gamma(v_2) \Rightarrow$$

$$v_2 m_2 \gamma(v_2) = \frac{\cancel{3}}{20} M \cdot \frac{4}{\cancel{5}} c \cdot \frac{\cancel{5}}{\cancel{3}} = \frac{1}{5} Mc \Rightarrow$$

$$\Rightarrow v_2 \cdot \frac{3}{4} M = \frac{1}{5} Mc \Rightarrow \boxed{v_2 = \frac{4}{15} c = 0.267c}$$

$$\gamma(v_2) = \frac{1}{\sqrt{1 - 0.267^2}} = 1.0377$$

$$m_2 = \frac{3}{4} \frac{M}{\gamma(v_2)} \Rightarrow \boxed{m_2 = 0.723 M}$$

(b) Initial mass M. Final mass : $m_1 + m_2 = 0.873 M$

$$\boxed{\Delta M = M - (m_1 + m_2) = 0.127 M}$$

$$\boxed{\text{total kinetic energy} = \Delta M c^2 = 0.127 Mc^2}$$

Check: $K_1 = (\gamma_1 - 1) m_1 c^2 = \frac{2}{3} \cdot \frac{3}{20} Mc^2 = 0.1 Mc^2$

$K_2 = (\gamma_2 - 1) m_2 c^2 = 0.0377 \times 0.723 Mc^2 = 0.0273 Mc^2$

$K_1 + K_2 = 0.127 Mc^2$ ✓

Problem 2

$hf = \frac{hc}{\lambda}$ is the energy of the photon.

For electrons to be emitted, their energy has to change by more than the work function ϕ. So the cutoff wavelength is

λ_{max}, so that $\frac{hc}{\lambda_{max}} = \phi = \frac{1240\, eV\, nm}{500\, nm} = \boxed{2.48\, eV}$

Stopping potential is $eV_s = \frac{hc}{\lambda} - \phi$

with $\lambda = 400\, nm$, $eV_s = \frac{1240}{400} eV - 2.48\, eV = 0.62\, eV$

$\Rightarrow \boxed{V_s = 0.62\, V}$

Problem 3

$$\lambda_m T = 2.8978 \times 10^{-3}\ m.K$$

$$\lambda_m = 1600\,nm \Rightarrow T = \frac{2.8978 . 10^{-3}\ mK}{1600 \times 10^{-9}\ m} = \boxed{1811\ K} \quad (a)$$

(b) From Stefan law, the power emitted by body of surface area A is

$$P = IA = \sigma T^4 A \quad , \text{with } \sigma = 5.67 \times 10^{-8}\ W/m^2K^4$$

The surface area of cylinder, ignoring top and bottom surfaces, is

$$A = 2\pi R . L, \quad \text{with } R = \text{radius},\ L = \text{length} . \quad \text{So}$$

$$P = \sigma T^4 . 2\pi R . L \Rightarrow L = \frac{P}{\sigma T^4 . 2\pi R}$$

$$L = \frac{60 W . m^2 . K^4}{5.67 . 10^{-8}\ W . 1811^4 . 2\pi . 10^{-3}\ m . K^4} = 0.0157\ m$$

$$\Rightarrow \boxed{L = 1.57\ cm}$$

Problem 4

Kinetic energy of electron $K = 17,150\,eV$; $\lambda' = 0.01\,nm$

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda'} \Rightarrow \frac{hc}{\lambda} = K + \frac{hc}{\lambda'} \Rightarrow$$

$$\Rightarrow \lambda = \frac{hc}{K + \frac{hc}{\lambda'}} = \frac{1240\,eV.nm}{17,150\,eV + \frac{1240\,eV.nm}{0.01\,nm}} = 0.00878\,nm$$

$$\boxed{\lambda = 0.00878\,nm}$$

(b) From $\lambda' - \lambda = \lambda_c(1 - \cos\theta) \Rightarrow 1 - \cos\theta = \frac{\lambda' - \lambda}{\lambda_c}$

$$\Rightarrow \cos\theta = 1 - \frac{\lambda' - \lambda}{\lambda_c}, \quad \lambda_c = 0.00243\,nm \Rightarrow$$

$$\cos\theta = 0.5 \Rightarrow \boxed{\theta = 60^\circ}$$

Problem 5

$$u(\lambda, T) = N(\lambda)\, E_{av}(\lambda, T)$$

$$N(\lambda) = \frac{2L}{\lambda^2}, \quad E_{av} = \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1} \Rightarrow$$

$$u(\lambda, T) = \frac{2L}{\lambda^3} \frac{hc}{e^{hc/\lambda k_B T} - 1}$$

The (total) intensity is proportional to $u = \int_0^\infty d\lambda\, u(\lambda)$

$$u = 2Lhc \int_0^\infty d\lambda \frac{1}{\lambda^3} \frac{1}{e^{hc/\lambda k_B T} - 1}$$

Change variables: $\frac{hc}{\lambda k_B T} = x \Rightarrow \frac{hc}{\lambda^2 k_B T} d\lambda = dx \Rightarrow$

$$\Rightarrow \frac{d\lambda}{\lambda^2} = \frac{k_B T}{hc} dx, \quad \text{and} \quad \frac{1}{\lambda} = \frac{k_B T}{hc} x \Rightarrow \frac{d\lambda}{\lambda^3} = \frac{(k_B T)^2}{(hc)^2} dx \cdot x$$

$$\Rightarrow u = 2Lhc \frac{(k_B T)^2}{(hc)^2} \int_0^\infty dx \frac{x}{e^x - 1}$$

therefore,

$$I \propto u \propto T^2 \Rightarrow \boxed{n = 2}$$