Physics 2D Quiz 1 Solutions Winter 2025

Problem 1

$\Delta t = 1\mu s$, $\Delta t = 1.1\mu s$

According to relativity, $\sqrt{1-u^2/c^2} = \frac{1}{1.1}$, (time dilation), where u is the speed of the train.

Solving for $\frac{u}{c}$: $1-\frac{u^2}{c^2} = \frac{1}{1.1^2} \Rightarrow \frac{u^2}{c^2} = 1-\frac{1}{1.1^2} \Rightarrow$

$$\boxed{\frac{u}{c} = \sqrt{1-\frac{1}{1.1^2}} = 0.4166} \qquad \boxed{\gamma = \frac{1}{\sqrt{1-u^2/c^2}} = 1.1}$$

According to observers on the train, you are moving at speed u relative to them, and take time $1.1\mu s$ to go from front to back $\Rightarrow$

$$L_0 = u \cdot 1.1\mu s = \frac{u}{c} \cdot c \cdot 1.1\mu s = 0.4166 \times 3\times10^8 \times 1.1\times10^{-6}\, m$$

$$\Rightarrow \boxed{L_0 = 137\,m}$$

Alternative solution:

The length of the train as measured by observers on the ground is

$$L = u \cdot 1\mu s = 125m = 0.4166 \cdot 3\times10^8 \cdot 10^{-6}\, m$$

it is contracted because the train is moving.

$$L = \frac{L_0}{\gamma} \Rightarrow L_0 = L\cdot\gamma = \frac{L}{\sqrt{1-u^2/c^2}} = 1.1L = 137m.$$

Problem 2

event "chicken is born" : (x_c, t_c) on ground
(x'_c, t'_c) on ship

event "hen lays egg" : (x_e, t_e) on ground
(x'_e, t'_e) on ship

simultaneous on ~~earth~~ ground $\Rightarrow t_c = t_e$

$t' = \gamma(t - \frac{u x}{c^2})$ Lorentz $\Rightarrow$

$t'_c = \gamma(t_c - \frac{u x_c}{c^2})$

$t'_e = \gamma(t_e - \frac{u x_e}{c^2})$

$t'_c - t'_e = \frac{\gamma u}{c^2}(x_e - x_c)$

$x_e - x_c > 0$ (egg is in front, chicken in back) $\Rightarrow \boxed{t'_c > t'_e}$

$\Rightarrow$ according to observers on ship, egg came first, chicken later.

The proper length is $L_0 = 500\,m$. As seen from earth ~~it~~ the length of the ship is $x_e - x_c = \frac{L_0}{\gamma}$ (length contraction) $\Rightarrow$

$$t'_c - t'_e = \frac{\gamma u}{c^2}\frac{L_0}{\gamma} = \frac{u}{c}\frac{L_0}{c} = \frac{0.6 \times 500\,m}{3 \times 10^8\,m} \cdot s = 10^{-6} s$$

$$\Rightarrow \boxed{t'_c - t'_e = 1\,\mu s}$$

Alternative solution : use $t = \gamma(t' + u x'/c^2)$

$t_c = \gamma(t'_c + u x'_c/c^2) = t_e = \gamma(t'_e + u x'_e/c^2) \Rightarrow$

$\Rightarrow t'_c - t'_e = \frac{u}{c^2}(x'_e - x'_c) = \frac{u}{c^2} L_0$ same answer

Problem 3

initially	later	even later
A $\longrightarrow$ 0.4c	A $\rightarrow$ 0.4c	A $\rightarrow$
B	B $\rightarrow$ 0.8c	B

(a) relative speed of B with respect to A:

putting O' frame on A, $u = 0.4c$, $v_A = v = 0.8c$

$$v_{BA} = \frac{v_A - u}{1 - \frac{u v_A}{c^2}} = \frac{0.8c - 0.4c}{1 - 0.4 \cdot 0.8} = 0.588c$$

$\boxed{v_{BA} = 0.588c}$ (a)

(b) When twin A turns 21, B is at a distance

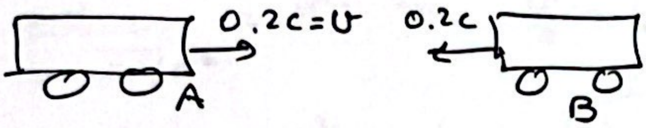
$D = 0.4c \times 1\,\text{year}$ as seen in A frame.

B is approaching A at speed v_{BA}, to travel distance D the time is

$$\Delta t = \frac{D}{v_{BA}} = \frac{0.4c \times 1\,\text{year}}{0.588c} = 0.68\,\text{years}.$$

So twin A is $\boxed{21.68 \text{ years old}}$ when B catches up. (b)

Problem 4



relative speed

$$v_{rel} = \frac{2v}{1+\frac{v^2}{c^2}} = \frac{2v}{1+0.2^2} = 1.923v = \boxed{0.3846c}$$

Doppler shift:

$$f' = f\sqrt{\frac{1+v_{rel}/c}{1-v_{rel}/c}} \Rightarrow \lambda' = \lambda\sqrt{\frac{1-v_{rel}/c}{1+v_{rel}/c}}$$

Driver of car A sees that light emitted from car B is blue $\Rightarrow$

$$\Rightarrow \quad 500\,nm = \lambda_B\sqrt{\frac{1-v_{rel}/c}{1+v_{rel}/c}}$$

$$\Rightarrow \quad \lambda_B = 500\,nm\sqrt{\frac{1+v_{rel}/c}{1-v_{rel}/c}} = 500\,nm \times 1.5$$

$$\Rightarrow \quad \boxed{\lambda_B = 750\,nm} \quad (a)$$

(b) B sees the light emitted from A ~~wi~~ having wavelength

$$\boxed{\lambda_A(\text{seen by } B) = \frac{500\,nm}{1.5} = 333.3\,nm}$$

Alternative solution

Wavelength of B light as seen by observer on the ground:

$$\lambda_{B,ground} = \lambda_B\sqrt{\frac{1-v/c}{1+v/c}}$$

Observer on ground absorbs and emits that light. Observer in A sees

$$\lambda_B(\text{seen from } A) = \lambda_{B,ground}\sqrt{\frac{1-v/c}{1+v/c}} = \lambda_B\,\frac{1-v/c}{1+v/c} = \lambda_B\cdot\frac{0.8}{1.2} = \frac{\lambda_B}{1.5}$$

Since $\lambda_B(\text{seen from } A) = 500\,nm$

$$\Rightarrow \quad \boxed{\lambda_B = 1.5 \times 500\,nm = 750\,nm}$$

Problem 5

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} = mc \Rightarrow \frac{v}{c} = \sqrt{1-v^2/c^2} \Rightarrow$$

$$\frac{v^2}{c^2} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{1}{2} \Rightarrow \boxed{\frac{v}{c} = \frac{1}{\sqrt{2}} = 0.707}$$

(b) $K = (\gamma - 1) m c^2$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{v/c} = \sqrt{2}$$

$$\Rightarrow \boxed{K = (\sqrt{2}-1) mc^2 = 0.414\, mc^2}$$