

Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$; $v'_y = \frac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \rightarrow -u$, primed \leftrightarrow unprimed; Doppler: $f' = f \sqrt{\frac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$
 $E = \sqrt{p^2 c^2 + m^2 c^4}$; rest energy: $E_0 = mc^2$

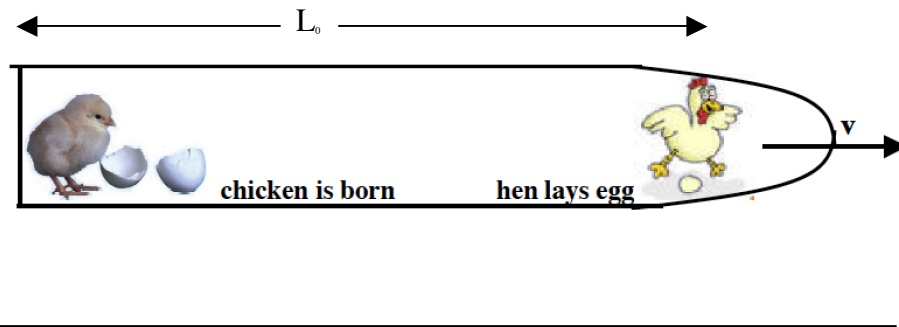
Electron: $m_e = 0.511 \text{ Mev}/c^2$; Proton: $m_p = 938.26 \text{ Mev}/c^2$; Neutron: $m_n = 939.55 \text{ Mev}/c^2$

Problem 1 (6 points)

A long train is passing by you. According to your stopwatch it takes $1 \mu\text{s}$ for the train to pass by you. Observers in the train see you pass by the front and the back of the train in a time interval $1.1 \mu\text{s}$ as measured in the train.

What is the proper length of this train? Give your answer in meters. Justify your answer.

Problem 2 (6 points)



At the front end of a spaceship of length $L_0 = 500 \text{ m}$ (as measured in the spaceship) a hen lays an egg. At the back end, a chicken is born. The ship is moving at speed $v = 0.6c$ with respect to the ground. According to observers on the ground, the two events (chicken is born, hen lays egg) happened simultaneously.

(a) According to observers on the spaceship which came first, the chicken or the egg? Justify your answer.

(b) According to observers on the spaceship what was the time difference between both events? Give the answer in μs ($1 \mu\text{s} = 10^{-6} \text{ s}$). Justify your answer.

Hint: use Lorentz transformation.

Problem 3 (6 points)

Twin A and twin B own spaceships. Twin A's spaceship travels at speed $0.4c$, twin B's spaceship travels at speed $0.8c$.

On their 20th birthday, twin A departs earth on its spaceship towards a distant star. Twin B promises to leave later and eventually catch up, since its spaceship is faster.

When twin A turns 21, as measured in its own reference frame, twin B departs. Assume the events "twin A turns 21" and "B departs" are simultaneous in twin A's reference frame.

- How fast does twin B travel relative to twin A?
- How old is twin A when twin B catches up with twin A, as measured in twin A's reference frame?

Problem 4 (6 points)

Cars A and B are moving towards each other, each moving at speed $0.2c$ with respect to the ground. Car A emits blue light, of wavelength $\lambda_{blue} = 500nm$. The driver of car A sees that the light emitted from car B is also blue, with the same wavelength.

- What is the wavelength of the light emitted by car B as seen by car B's driver?
- What is the wavelength of the light emitted by car A as seen by car B's driver?

Hint: $\lambda = c / f$

Problem 5 (6 points)

A particle of mass m has momentum $p=mc$

- Find its speed v . Give your answer as v/c .
- Find its kinetic energy. Give your answer as a number times mc^2 .