15. The shortest wavelength is the series limit. For the Lyman series, $n_0 = 1$ and Equation 6.21 becomes

$$\lambda = (91.13\ \text{nm})\frac{n^2}{n^2 - 1} \qquad n = 2, 3, 4, \ldots$$

which gives λ = 121.51 nm (n = 2), 102.52 nm (n = 3), 97.21 nm (n = 4).

18. From Figure 6.16 we see that only the Paschen series ($n_0 = 3$) has lines near 1000 nm. Using the series limit of 820.1 nm, we have from Eq. 6.21

$$1005\ \text{nm} = (820.1\ \text{nm})\frac{n^2}{n^2 - 9}$$

$$1.225(n^2 - 9) = n^2$$

Solving, we find $n = 7$, so the transition connects the $n = 7$ and $n = 3$ states.

19. $r_3 = 9a_0 = 9(0.0529\text{ nm}) = 0.476\text{ nm}$

$$v = \frac{n\hbar}{mr} = c\frac{n\hbar c}{mc^2 r} = c\frac{3(1240\,\text{eV}\cdot\text{nm})/2\pi}{(0.511\times10^6\text{ eV})(0.476\,\text{nm})} = 2.43\times10^{-3}c = 7.30\times10^5\text{ m/s}$$

$$U = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{1.440\text{ eV}\cdot\text{nm}}{0.476\text{ nm}} = -3.02\text{ eV}$$

$$K = \frac{e^2}{8\pi\varepsilon_0}\frac{1}{r} = \frac{1.440\,\text{eV}\cdot\text{nm}}{2(0.476\text{ nm})} = 1.51\,\text{eV}$$

21. (a) From Equation 6.26, $v = \frac{n\hbar}{mr} = \frac{n\hbar}{mn^2 a_0}$. Using Equation 6.29 for a_0, we obtain

$$v = \frac{\hbar}{nm(4\pi\varepsilon_0 \hbar^2 / me^2)} = \frac{e^2}{4\pi\varepsilon_0} \frac{1}{n\hbar} = \frac{\alpha c}{n}$$

(b) When the nuclear charge is Ze, we must replace e^2 with Ze^2, so $v = Z\alpha c/n$.

22. The energy of the initial $n = 5$ state is $E_5 = \dfrac{-13.6\text{ eV}}{25} = -0.544\text{ eV}$. An electron in this state can make transitions to any of the lower states with $n = 4$ ($E_4 = -0.850$ eV), $n = 3$

(E_3 = –1.51 eV), n = 2 (E_2 = –3.40 eV), and n = 1 (E_1 = –13.6 eV). The transition energies are:

$$5 \to 4: \quad \Delta E = E_5 - E_4 = -0.544 \text{ eV} - (-0.850 \text{ eV}) = 0.306 \text{ eV}$$
$$5 \to 3: \quad \Delta E = E_5 - E_3 = -0.544 \text{ eV} - (-1.51 \text{ eV}) = 0.97 \text{ eV}$$
$$5 \to 2: \quad \Delta E = E_5 - E_2 = -0.544 \text{ eV} - (-3.40 \text{ eV}) = 2.86 \text{ eV}$$
$$5 \to 1: \quad \Delta E = E_5 - E_1 = -0.544 \text{ eV} - (-13.6 \text{ eV}) = 13.1 \text{ eV}$$

24. The photon energy of the incident light is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{53.0\ \text{nm}} = 23.4\ \text{eV}$$

When an atom in the ground state absorbs a 23.4-eV photon, the atom is ionized (which takes 13.6 eV). The excess energy, 23.4 eV – 13.6 eV = 9.8 eV, appears as the kinetic energy of the electron, which is now free of the atom. Neglecting a small recoil kinetic energy given to the proton, the electrons have a kinetic energy of 9.8 eV.

25. (a) The ionization energy is the magnitude of the energy of the electron. For the $n = 3$ level of hydrogen

$$|E_3| = \left|\frac{-13.6\text{ eV}}{9}\right| = 1.51\text{ eV}$$

(b) For singly ionized helium ($Z = 2$) we use Equation 6.38:

$$|E_n| = \left|\frac{(-13.6\text{ eV})Z^2}{n^2}\right| = \left|\frac{(-13.6\text{ eV})2^2}{2^2}\right| = 13.6\text{ eV}$$

(c)

$$|E_n| = \left|\frac{(-13.6\text{ eV})Z^2}{n^2}\right| = \left|\frac{(-13.6\text{ eV})3^2}{4^2}\right| = 7.65\text{ eV}$$

27. The Lyman series consists of transitions that end in the $n = 1$ level. The smallest energy difference, corresponding to the longest wavelength, is $n = 2$ to $n = 1$.

$$\Delta E = E_2 - E_1 = (-13.6\text{ eV})2^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 40.8\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{ eV}\cdot\text{nm}}{40.8\text{ eV}} = 30.4\text{ nm}$$

The largest energy difference would correspond to transitions from $n = \infty$ to $n = 1$:

$$\Delta E = E_\infty - E_1 = (-13.6\text{ eV})2^2\left(0 - \frac{1}{1^2}\right) = 54.4\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{ eV}\cdot\text{nm}}{54.4\text{ eV}} = 22.8\text{ nm}$$

28. Using Equation 6.38, we have $E_n = (-13.6 \text{ eV})Z^2/n^2 = (-54.4 \text{ eV})/n^2$, so $E_1 = -54.40$ eV, $E_2 = -13.60$ eV, $E_3 = -6.04$ eV, $E_4 = -3.40$ eV. The possible transitions are:

$$4 \to 1: \quad \Delta E = E_4 - E_1 = 51.00 \text{ eV} \qquad \lambda = hc/\Delta E = 24.31 \text{ nm}$$

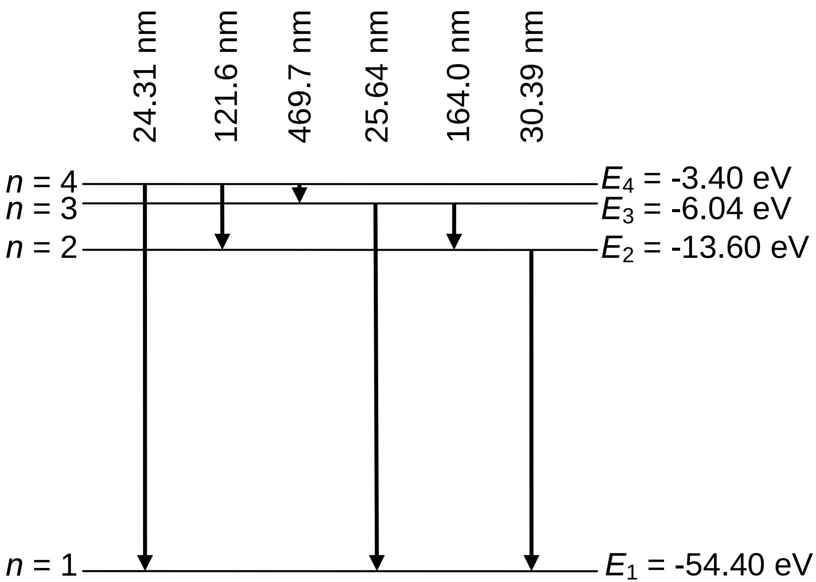
$$4 \to 2: \quad \Delta E = E_4 - E_2 = 10.20 \text{ eV} \qquad \lambda = hc/\Delta E = 121.6 \text{ nm}$$

$$4 \to 3: \quad \Delta E = E_4 - E_3 = 2.64 \text{ eV} \qquad \lambda = hc/\Delta E = 469.7 \text{ nm}$$

$$3 \to 1: \quad \Delta E = E_3 - E_1 = 48.36 \text{ eV} \qquad \lambda = hc/\Delta E = 25.64 \text{ nm}$$

$$3 \to 2: \quad \Delta E = E_3 - E_2 = 7.56 \text{ eV} \qquad \lambda = hc/\Delta E = 164.0 \text{ nm}$$

$$2 \to 1: \quad \Delta E = E_2 - E_1 = 40.80 \text{ eV} \qquad \lambda = hc/\Delta E = 30.39 \text{ nm}$$



30. (a) If the circumference is an integral number of de Broglie wavelengths ($2\pi r = n\lambda$), then after each orbit the waves will align, peak to peak and valley to valley, to give standing waves.

(b) $2\pi r = n\lambda = n\dfrac{h}{p} = \dfrac{nh}{mv}$ so $mvr = \dfrac{nh}{2\pi} = n\hbar$

35. (a) The frequency of revolution is given by Equation 6.41:

$$f_n = \frac{me^4}{32\pi^3\varepsilon_0^2\hbar^3}\frac{1}{n^3} = \frac{1}{\pi\hbar}\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^3} = \frac{13.6\text{ eV}}{\pi\hbar}\frac{1}{n^3} = \frac{6.58\times10^{15}\text{ Hz}}{n^3}$$

A similar calculation gives the radiation frequency from Equation 6.42:

$$f = \frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3}\frac{2n-1}{n^2(n-1)^2} = \frac{13.6\text{ eV}}{2\pi\hbar}\frac{2n-1}{n^2(n-1)^2} = (6.58\times10^{15}\text{ Hz})\frac{2n-1}{2n^2(n-1)^2}$$

For n = 10, we get $f_n = 6.58\times10^{12}$ Hz and $f = 7.72\times10^{12}$ Hz.

(b) For n = 100, $f_n = 6.58\times10^{9}$ Hz and $f = 6.68\times10^{9}$ Hz.

(c) For n = 1000, $f_n = 6.58\times10^{6}$ Hz and $f = 6.59\times10^{6}$ Hz.

(d) For n = 10,000, $f_n = 6.58\times10^{3}$ Hz and $f = 6.58\times10^{3}$ Hz. Note how f approaches f_n as n becomes large, in accordance with the correspondence principle.

36. The Rydberg constant in ordinary hydrogen is

$$R_{\text{H}} = R_\infty\left(1+\frac{m}{M_{\text{H}}}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\text{ u}}{1.007825\text{ u}}\right) = R_\infty(1.000544)$$

and in "heavy" hydrogen or deuterium:

$$R_D = R_\infty\left(1+\frac{m}{M_D}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\ \text{u}}{2.104102\ \text{u}}\right) = R_\infty(1.000272)$$

From Equation 6.33 the difference in wavelengths for the first line of the Balmer series (n = 3 to n = 2) is

$$\lambda_D - \lambda_H = \left(\frac{1}{R_D} - \frac{1}{R_H}\right)\left(\frac{3^2 2^2}{3^2 - 2^2}\right) = \frac{7.2}{1.09737\times10^7\ \text{m}^{-1}}\left(\frac{1}{1.000272} - \frac{1}{1.000544}\right) = 0.178\ \text{nm}$$