

$$23. \quad (a) \quad E_0 = \frac{1}{2} \hbar \omega_0 = \frac{1}{2} k x_0^2 \quad \text{so} \quad x_0 = \sqrt{\hbar \omega_0 / k}$$

$$(b) \quad E_1 = \frac{3}{2} \hbar \omega_0 = \frac{1}{2} k x_0^2 \quad \text{so} \quad x_0 = \sqrt{3 \hbar \omega_0 / k}$$

$$E_2 = \frac{5}{2} \hbar \omega_0 = \frac{1}{2} k x_0^2 \quad \text{so} \quad x_0 = \sqrt{5 \hbar \omega_0 / k}$$

$$24. \quad x_{\text{av}} = \int_{-\infty}^{\infty} |\psi(x)|^2 x dx = A^2 \int_{-\infty}^{\infty} e^{-2ax^2} x dx = 0$$

because the integrand is an odd function of x (the integral from $-\infty$ to 0 exactly cancels the integral from 0 to $+\infty$).

$$(x^2)_{\text{av}} = \int_{-\infty}^{\infty} |\psi(x)|^2 x^2 dx = A^2 \int_{-\infty}^{\infty} e^{-2ax^2} x^2 dx = 2A^2 \int_0^{\infty} e^{-2ax^2} x^2 dx = \frac{2A^2}{\sqrt{8a^3}} \int_0^{\infty} e^{-u^2} u^2 du$$

with the substitution $u = x\sqrt{2a}$. The integral is a standard form found in tables and is equal to $\sqrt{\pi}/4$. Substituting $A = (\omega_0 m / \pi \hbar)^{1/4}$ and $a = \sqrt{km}/2\hbar = \omega_0 m / 2\hbar$, we find

$$(x^2)_{\text{av}} = 2 \left(\frac{\omega_0 m}{\pi \hbar} \right)^{1/2} \frac{1}{2\sqrt{2}} \left(\frac{2\hbar}{\omega_0 m} \right)^{3/2} \frac{\sqrt{\pi}}{4} = \frac{\hbar}{2\omega_0 m}$$

$$\Delta x = \sqrt{(x^2)_{\text{av}} - (x_{\text{av}})^2} = \sqrt{\hbar / 2m\omega_0}$$

25. (a) Because the oscillating particle moves with equal probability in the positive and negative x directions, $p_{\text{av}} = 0$.

$$(b) \quad U_{\text{av}} = \frac{1}{2} k (x^2)_{\text{av}} = \frac{1}{2} k \frac{\hbar}{2\omega_0 m} = \frac{1}{2} \omega_0^2 m \frac{\hbar}{2\omega_0^2 m} = \frac{1}{4} \hbar \omega_0$$

$$K_{\text{av}} = E - U_{\text{av}} = \frac{1}{2} \hbar \omega_0 - \frac{1}{4} \hbar \omega_0 = \frac{1}{4} \hbar \omega_0$$

$$(p^2)_{\text{av}} = 2mK_{\text{av}} = 2m \left(\frac{1}{4} \hbar \omega_0 \right) = \frac{\hbar \omega_0 m}{2}$$

$$(c) \quad \Delta p = \sqrt{(p^2)_{\text{av}} - (p_{\text{av}})^2} = \sqrt{\hbar\omega_0 m/2}$$

$$26. \quad E_0 = 1.24 \text{ eV} = \frac{1}{2}\hbar\omega_0 \quad \text{so} \quad \hbar\omega_0 = 2.48 \text{ eV}$$

$$\text{To } n = 2 \text{ state:} \quad \Delta E = E_2 - E_0 = \frac{5}{2}\hbar\omega_0 - \frac{1}{2}\hbar\omega_0 = 2\hbar\omega_0 = 2(2.48 \text{ eV}) = 4.96 \text{ eV}$$

$$\text{To } n = 4 \text{ state:} \quad \Delta E = E_4 - E_0 = \frac{9}{2}\hbar\omega_0 - \frac{1}{2}\hbar\omega_0 = 4\hbar\omega_0 = 4(2.48 \text{ eV}) = 9.92 \text{ eV}$$

$$27. \quad P(x) dx = |\psi(x)|^2 dx = A^2 e^{-2ax^2} dx \quad \text{so at } x = 0 \quad P(0)dx = A^2 dx$$

At the classical turning points $x = \pm x_0$, $K = 0$ so $E = U$ or $\frac{1}{2}\hbar\omega_0 = \frac{1}{2}kx_0^2$

$$P(\pm x_0)dx = A^2 e^{-2(\sqrt{km}/2\hbar)(\hbar\omega_0/k)} dx = A^2 e^{-1} dx = e^{-1} P(0)dx = 0.368 P(0)dx$$

28. (a) If $E = 0$, then $p = 0$ and we would know the momentum exactly. Thus $\Delta p = 0$, which means $\Delta x = \infty$. But that would be inconsistent with a particle that is bound to a finite region of space.

(b)

$$E = \frac{1}{2}\hbar\omega_0 = \frac{1}{2}\hbar\sqrt{\frac{k}{m}} = \frac{1}{2}\hbar c\sqrt{\frac{k}{mc^2}} = 0.5(197 \text{ eV} \cdot \text{nm})\sqrt{\frac{3.5 \times 10^3 \text{ eV/nm}^2}{938 \times 10^3 \text{ eV}}} = 0.19 \text{ eV}$$

This is less than the binding energy, so this motion is not sufficient to dissociate the molecule.

(c) At the turning point of the motion, $E = \frac{1}{2}kx_0^2$, so

$$x_0 = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.19 \text{ eV})}{3.5 \times 10^3 \text{ eV/nm}^2}} = 0.010 \text{ nm}$$

This motion is not negligible at the atomic level.

$$32. \quad x < 0: \quad \psi_0 = A'e^{ik_0x} + B'e^{-ik_0x} \quad \text{with} \quad k_0 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$x > 0: \quad \psi_1(x) = C'e^{ik_1x} + D'e^{-ik_1x} \quad \text{with} \quad k_1 = \sqrt{\frac{2m(E - U_0)}{\hbar^2}}$$

If the particles are incident from the negative x direction, then D' (which is the coefficient of the term that represents a wave in the region of positive x traveling toward the origin) must be set to 0. We then apply the continuity conditions on ψ and $d\psi/dx$ at $x = 0$:

$$\psi_0(0) = \psi_1(0): \quad A' + B' = C'$$

$$\left(\frac{d\psi_0}{dx}\right)_{x=0} = \left(\frac{d\psi_1}{dx}\right)_{x=0} : \quad k_0(A' - B') = k_1C'$$

Solving these two equations, we obtain

$$C' = \frac{2A'}{1 + k_1/k_0}$$

$$B' = C' - A' = \frac{2A'}{1 + k_1/k_0} - A' = \frac{1 - k_1/k_0}{1 + k_1/k_0} A'$$

The squares of the amplitude ratios give the relative probabilities for the particles to be reflected at $x = 0$ or transmitted into the $x > 0$ region:

Reflection probability: $\frac{|B'|^2}{|A'|^2} = \left(\frac{1 - k_1/k_0}{1 + k_1/k_0} \right)^2$

Transmission probability: $\frac{|C'|^2}{|A'|^2} = \frac{4}{(1 + k_1/k_0)^2}$

42. (a) The x and y motions are independent, and each contributes an energy of $\hbar\omega_0(n + \frac{1}{2})$, but the integer n is not necessarily the same for the two independent motions. Thus the total energy is

$$E = \hbar\omega_0(n_x + \frac{1}{2}) + \hbar\omega_0(n_y + \frac{1}{2}) = \hbar\omega_0(n_x + n_y + 1)$$

(b)

$4\hbar\omega_0$		4	(0,3), (1,2), (2,1), (3,0)
$3\hbar\omega_0$		3	(0,2), (1,1), (2,0)
$2\hbar\omega_0$		2	(0,1), (1,0)
$\hbar\omega_0$		1	(0,0)
Energy		Degeneracy	(n_x, n_y)

- (c) The level with energy $N\hbar\omega_0$ has N different possible sets of quantum numbers n_x, n_y . Both n_x and n_y range from 0 to $N-1$ but with their sum fixed to N . The number of possible values of n_x is then N (the values are 0, 1, 2, ..., $N-2$, $N-1$), and for each value of n_x the value of n_y is fixed. The total degeneracy of each level is thus $N = n_x + n_y + 1$.