PHYSICS 2D QUIZ 5 FALL QUARTER 2023
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Justify all your answers to all 5 problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3\times10^8\, m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6\times10^{-19} J$

Photoelectric effect: $eV_s = K_{max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037\times10^{-8} W/m^2\cdot K^4$; Wien's law: $\lambda_m T = 2.8978\times10^{-3}\, m\cdot K$

$I(T) = \int_0^\infty I(\lambda,T)\, d\lambda$; $I = (c/4)u$; $u(\lambda,T) = N(\lambda)E_{av}(\lambda,T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\,dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\,dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240\, eV\cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604)\, eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d\sin\theta = n\lambda$; single-slit diffraction minima: $a\sin\theta = n\lambda$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x \Delta k \sim 1$ $\Delta t \Delta\omega \sim 1$; $\Delta x \Delta p \sim \hbar$ $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

$\hbar c = 197.3$ eV nm ; group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\, \psi^*\psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $< f(x) >= \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 0.0381 eVnm^2$ (electron)

2D square well: $\Psi_{n_1n_2}(x,y) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $E_{n_1n_2} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}}\sin(\frac{n\pi w}{L_i})$

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2$

Step potential: reflection coef : $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Rutherford scattering: $U = \frac{2Ze^2}{4\pi\varepsilon_0 r}$; $e^2/(4\pi\varepsilon_0) = 1.44 eV \cdot nm = ke^2$

$b = \frac{Z}{K_\alpha}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$; $f_{>\theta} = nt\pi b^2$; $N(\theta) = \text{constant} \times (\frac{Z}{K_\alpha})^2 \times \frac{1}{\sin^4(\theta/2)}$

Line spectra: $\frac{1}{\lambda} = R(\frac{1}{n_0^2} - \frac{1}{n^2})$; $R = \frac{1}{91.13nm}$

Bohr atom: $r_n = (a_0/Z)n^2$; $E_n = -E_0Z^2/n^2$; $E_0 = \frac{ke^2}{2a_0}$; $a_0 = \frac{\hbar^2}{m_eke^2}$; $L = m_evr = n\hbar$

$E_0 = 13.6eV$; $a_0 = 0.0529nm$

Observables, operators, eigenvalues, eigenfunctions:
$< A >= \int d^3r\psi^*(\vec{r})A_{op}\psi(\vec{r})$; if $A_{op}\psi = a\psi$ ==> $\Delta A = 0$

Spherically symmetric potential: $\Psi_{n,\ell,m_\ell}(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell,m_\ell}(\theta,\phi)$; $Y_{\ell,m_\ell}(\theta,\phi) = P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

quantum numbers: $n = 1,2,3,...$; $0 \le \ell \le n-1$; $-\ell \le m_\ell \le \ell$

Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$; $|L^2| = \ell(\ell+1)\hbar^2$; $L_z = m_\ell \hbar$

Radial probability density: $P(r) = r^2 |R_{n\ell}(r)|^2$; Energy: $E_n = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$

Ground state of hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2} e^{-Zr/a_0}$; $\int_0^\infty dr\, r^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$

Orbital magnetic moment: $\vec{\mu} = \frac{-e}{2m_e}\vec{L}$; $\mu_z = -\mu_B m_l$; $\mu_B = \frac{e\hbar}{2m_e} = 5.79 \times 10^{-5} eV/T$

Spin 1/2: $s = \frac{1}{2}$, $|\vec{S}| = \sqrt{s(s+1)}\hbar$; $S_z = m_s \hbar$; $m_s = \pm 1/2$; $\vec{\mu}_s = \frac{-e}{2m_e} g\vec{S}$; $g = 2$

Orbital+spin mag moment: $\vec{\mu} = \frac{-e}{2m_e}(\vec{L} + g\vec{S})$; Energy in mag. field: $U = -\vec{\mu} \cdot \vec{B}$

Problem 1 (6 points)
Electromagnetic radiation with uniform intensity in the wavelength range $100nm < \lambda < 200nm$ is incident on a dilute gas of hydrogen atoms whose electrons are in the ground state initially. The atoms absorb and subsequently emit radiation. List all the possible values of the wavelength of the emitted radiation, in nm.

Problem 2 (6 points)

An electron is in a Bohr orbit of a hydrogen-like ion with nuclear charge Ze that has radius 0.0529 nm and angular momentum $3\hbar$.
(a) Find its speed v. Give the answer as v/c.
(b) Find its kinetic energy, in eV.
(c) Find its potential energy, in eV.

Problem 3 (6 points)
The angular momentum operators discussed in class are given by the formulae

$$L^2_{op} = -\hbar^2[\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} + \frac{\partial^2}{\partial\theta^2} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}] \quad ; \quad L_{z,op} = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$$

An electron in a hydrogen atom is in a stationary state described by the wavefunction
$\psi(r,\theta,\phi) = R(r)(3\cos^2\theta - 1)$
By applying the operators given above to this wavefunction, find for this electron:
(a) Its quantum number m_ℓ.
(b) Its quantum number ℓ.
(c) What can you say about its quantum number n?
Justify your answers.

Problem 4 (6 points)
An electron in a hydrogen-like ion with nuclear charge Ze has radial wavefunction
$R(r) = Cr^3 e^{-r/(2a_0)}$ where C is a constant and a_0 is the Bohr radius, 0.0529 nm.

(a) Find the most probable radius for this electron in terms of a_0.
(b) Calculate (by doing integrals) the average potential energy for this electron. Give your answer as a number times Z, in eV units.
(c) Assuming that the most probable radius calculated in (a) is the same as what the Bohr model predicts, what is the Z for this ion? Justify your answer. Note: do not assume that the average potential energy is the same as in the Bohr model.

Problem 5 (6 points)
In the absence of a magnetic field, the ionization energy of hydrogen atoms in a gas is measured to be 13.5984 eV. When a magnetic field B is applied, it is found that the ionization energy for some atoms is 13.5974 eV and for some atoms it is 13.5994 eV. Explain why, and find how large B is, in T.