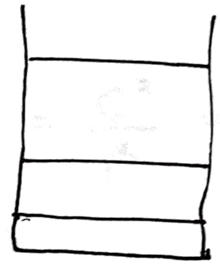
Physics 2D — Quiz 4 Solutions — Fall 2023

Problem 1



energies of well:

$$E_n = \frac{\hbar^2 \pi^2}{2 m_e L^2} n^2$$

for transitions between n_1 and n_2

$$\frac{hc}{\lambda} = E_{n_1} - E_{n_2} = \frac{\hbar^2 \pi^2}{2 m_e L^2}(n_1^2 - n_2^2)$$

photon has $\lambda = 1000\,nm$

So $$L^2 = \frac{\hbar \pi^2}{2 m_e} \frac{\lambda}{hc}(n_1^2 - n_2^2)$$

smallest L is when $n_1 = 2, n_2 = 1$, $n_1^2 - n_2^2 = 3$

$$L^2 = 0.0381\,eV\,nm^2 \cdot \pi^2 \times \frac{1000\,nm}{1240\,eV\,nm} \cdot 3 = 0.90975\,nm^2$$

$\boxed{L = 0.954\,nm}$ (a)

(b) The largest wavelength it can absorb is in going from $n=1$ to $n=2$, and it is of course $\boxed{\lambda = 1000\,nm}$

The next largest is in going from $n_1 = 2$ to $n_2 = 3$: $n_2^2 - n_1^2 = 9 - 4 = 5$

So: $$\frac{\lambda(2 \to 3)}{\lambda(1 - 2)} = \frac{3}{5} = 0.6 \Rightarrow \boxed{\lambda = 600\,nm}$$

Problem 2

$$\psi(x) = C e^{-\beta|x|} \qquad \beta = 4\,nm^{-1}$$

$$1 = \int_{-\infty}^{\infty} dx\, |\psi(x)|^2 = 2C^2 \int_0^{\infty} dx\, e^{-2\beta x} = \frac{2C^2}{2\beta} = \frac{C^2}{\beta} \Rightarrow$$

$$\Rightarrow \boxed{C = \beta^{1/2} = 2\,nm^{-1/2}} \quad (a)$$

$$P(-0.1 < x < 0.1) = 2P(0 < x < 0.1) = 2C^2 \int_0^{0.1} dx\, e^{-2\beta x} = \frac{C^2}{\beta}\left[1 - e^{-0.2\beta}\right]$$

$$= 1 - e^{-0.2\beta} = 1 - e^{-0.8} = \boxed{0.55} \quad (b)$$

(c) $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \beta^2 \psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\beta^2 + U(x) = E \Rightarrow U = E + \frac{\hbar^2}{2m}\beta^2$$

$$\Rightarrow U = 2eV + 0.0381\, eV\, nm^2 \times 16\, nm^{-2} = 2.61 eV$$

$$\Rightarrow \boxed{U(x = 0.5\, nm) = 2.61 eV}$$

Problem 3

(a) electrons in this potential can only emit photons of the same wavelength, i.e. 10,000 nm.

(b) Classical amplitude:

$$E = \frac{1}{2} m_e \omega^2 A^2 \quad ; \text{ in ground state } E = \frac{\hbar\omega}{2}$$

We know that the photon energy is $\hbar\omega = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{10{,}000\,nm} = 0.124\,eV$

$$\frac{\hbar\omega}{2} = \frac{1}{2} m_e \omega^2 A^2 \Rightarrow A^2 = \frac{\hbar \cancel{\omega}}{m_e \omega^{\cancel{2}}} = \frac{\hbar}{m_e\omega} = \frac{\hbar^2}{m_e \hbar\omega} = \frac{2\hbar^2}{2m_e \hbar\omega}$$

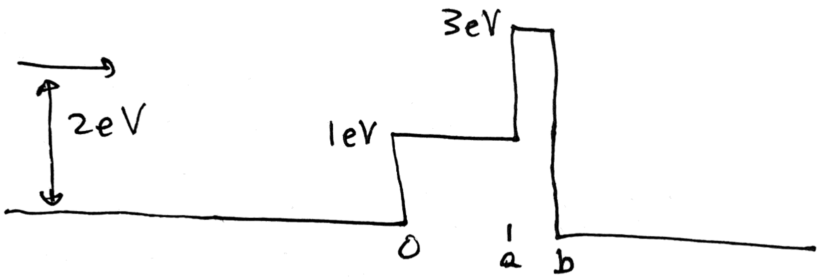
$$\Rightarrow \quad A^2 = 2 \times 0.0381 \frac{eV\,nm^2}{0.124\,eV} \Rightarrow \boxed{A = 0.784\,nm}$$

(c) $\Psi_0(x) = Ce^{-\frac{m\omega}{2\hbar}x^2} = Ce^{-\frac{1}{2A^2}x^2} \Rightarrow |\Psi_0(x)|^2 = C^2 e^{-\frac{x^2}{A^2}}$

$$\frac{|\Psi_0(A)|^2}{|\Psi_0(2A)|^2} = \frac{e^{-1}}{e^{-4}} = e^3 = 20.1$$

$\Rightarrow$ $\boxed{\text{it's} \sim 20 \text{ times more likely at } A \text{ than at } 2A}$

Problem 4



When electrons pass 0, they are partially reflected by this step:

$$R = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2 \qquad k_1 = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{2}{0.0381}}\ \text{nm}^{-1} = 7.245\ \text{nm}^{-1}$$

$$k_2 = \sqrt{\frac{2m(E - 1\text{eV})}{\hbar^2}} = \sqrt{\frac{1}{0.0381}}\ \text{nm}^{-1} = 5.123\ \text{nm}^{-1}$$

$$\Rightarrow \boxed{R = 0.172^2 = 0.0294}$$

So with 10,000 incoming electrons, 294 are reflected at $x = 0$,

9706 are transmitted.

When they encounter the barrier

$$T = e^{-2\alpha(b-a)} \qquad \alpha = \sqrt{\frac{2m(U-E)}{\hbar^2}} = \sqrt{\frac{2m}{\hbar^2}\cdot 1\text{eV}} = 5.123\ \text{nm}^{-1}$$

$b - a = 0.2\ \text{nm}$

$$\Rightarrow T = e^{-0.4 \times 5.123} = e^{-2.05} = \cancel{1.29}\ 0.129$$

So $9706 \times 0.129 =$ $\boxed{\text{1250 electrons reach the region } x > b}$

Problem 5

$$f_{>\theta} = n t \pi b^2 = C \cot^2\frac{\theta}{2}$$

so $$\frac{f_{>120°}}{f_{>90°}} = \frac{\cot^2 60°}{\cot^2 45°} = \frac{\cos^2 60° \ \cancel{\sin^2 45°}}{\sin^2 60° \cdot \cancel{\cos^2 45°}} = \left(\frac{1}{2}\right)^2 \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$

so approximately $100-33 = 67$ particles are scattered at angles between $90°$ and $120°$ in this experiment.

(b) closest approach d:

$$K_\alpha = \frac{2Ze^2}{4\pi\varepsilon_0 d} \Rightarrow d = \frac{2Z}{K_\alpha}\frac{e^2}{4\pi\varepsilon_0} = \frac{58}{5\times10^6\,eV} \cdot 1.44\,eV\,nm$$

$$\Rightarrow d = 16.7\times10^{-6}\,nm \Rightarrow \boxed{d = 16.7\,fm}$$