PHYSICS 2D QUIZ 4 FALL QUARTER 2023
PROF. HIRSCH

Justify all your answers to all 5 problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0/\gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m\vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6\times 10^{-19} J$

Photoelectric effect: $eV_s = K_{max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037\times 10^{-8} W/m^2\cdot K^4$; Wien's law: $\lambda_m T = 2.8978\times 10^{-3} m\cdot K$

$I(T) = \int_0^\infty I(\lambda,T)\,d\lambda$; $I = (c/4)u$; $u(\lambda,T) = N(\lambda)E_{av}(\lambda,T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\,dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\,dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240 eV\cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \lambda_c(1 - \cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d\sin\theta = n\lambda$; single-slit diffraction minima: $a\sin\theta = n\lambda$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x\Delta k \sim 1$ $\Delta t\Delta\omega \sim 1$; $\Delta x\Delta p \sim \hbar$ $\Delta t\Delta E \sim \hbar$; $\Delta A = \sqrt{<A^2> - <A>^2}$

$\hbar c = 197.3$ eV nm ; group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\, \psi^*\psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $< f(x) >= \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 0.0381 eVnm^2$ (electron)

2D square well: $\Psi_{n_1 n_2}(x,y) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $E_{n_1 n_2} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}}\sin(\frac{n\pi w}{L_i})$

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n + \frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$

Step potential: reflection coef : $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Rutherford scattering: $U = \frac{2Ze^2}{4\pi\varepsilon_0 r}$; $e^2/(4\pi\varepsilon_0) = 1.44 eV \cdot nm$

$b = \frac{Z}{K_\alpha}\frac{e^2}{4\pi\varepsilon_0}\cot\frac{1}{2}\theta$; $f_{>\theta} = nt\pi b^2$; $N(\theta) = \text{constant} \times (\frac{Z}{K_\alpha})^2 \times \frac{1}{\sin^4(\theta/2)}$

Line spectra: $\frac{1}{\lambda} = R(\frac{1}{n_0^2} - \frac{1}{n^2})$; $R = \frac{1}{91.13nm}$

Problem 1 (6 points)
An electron is in an excited state of a one-dimensional infinite potential well and makes a transition to a lower energy state, emitting a photon of wavelength 1000nm.
(a) What is the smallest possible length for this well, in nm?
(b) Assuming that length for the well, what are the two largest wavelength of photons an electron in that well can absorb in making a transition from a state to another state?

Problem 2 (6 points)
An electron is in a stationary state described by the wavefunction

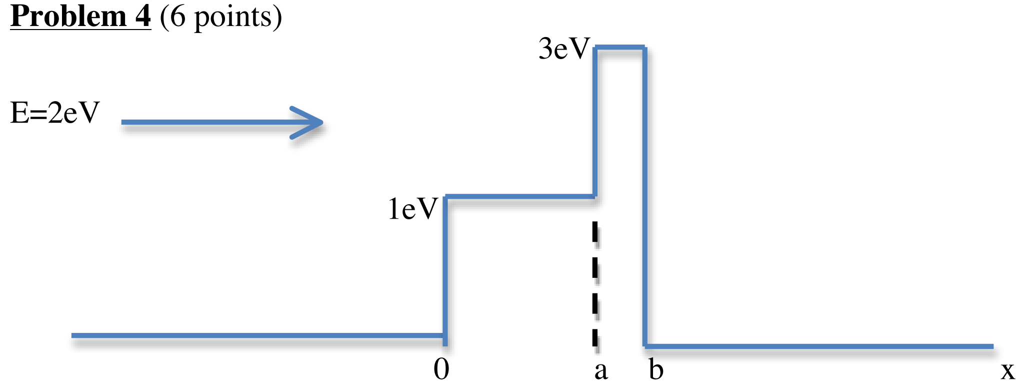
$\psi(x) = Ce^{-\beta x}$ for $x \geq 0$

$\psi(x) = Ce^{\beta x}$ for $x < 0$

with $\beta = 4nm^{-1}$. It has energy E=2eV
(a) Find C
(b) find the probability that the electron is in the region $-0.1nm < x < 0.1nm$
(c) The potential energy for this electron is U(x). Find U(x=0.5nm), in eV.

Problem 3 (6 points)
An electron moving in a harmonic oscillator potential can absorb photons only if their wavelength is 10,000 nm.
(a) What are the possible wavelengths of photons that this electron can emit?
(b) If this electron is in its ground state, what is its classical amplitude of oscillation?
(c) How much more likely is it to find this electron (in its ground state) at the classical amplitude versus at twice the classical amplitude?

Problem 4 (6 points)



A beam of 10,000 electrons with energy 2eV is incident from the left on the potential shown in the picture: U(x)=0 for x<0, U(x)=1eV for 0<x<a, U(x)=3eV for a<x<b, U(x)=0 for x>b, with a=2nm, b=2.2nm.
Find how many electrons approximately make it through into the region x>b.

Problem 5 (6 points)
In a Rutherford scattering experiment with a steady flow of α particles of kinetic energy 5MeV incident on a Copper foil (Z=29 for Cu) it is found that 100 α particles per second are scattered at angles greater than 90^0.
(a) How many α particles per second are scattered at angles between 90^0 and 120^0?
(b) What is the distance of closest approach of α particles to the center of nuclei in this experiment? Give your answer in $fm = 10^{-15} m$.