PHYSICS 2D QUIZ 3 FALL QUARTER 2023
PROF. HIRSCH

Justify all your answers to all 5 problems. Write clearly.

Time dilation; Length contraction: $\Delta t=\gamma\Delta t_0$; $L = L_0/\gamma$; $c = 3\times10^8\, m/s$

Lorentz transformation: $x' = \gamma(x-ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1-uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1-uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1-u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1\pm u/c}{1\mp u/c}}$

Momentum: $\vec{p} = \gamma m\vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma-1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6\times10^{-19} J$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, σ=$5.67037\times10^{-8} W/m^2\cdot K^4$; Wien's law: $\lambda_m T = 2.8978\times10^{-3}\, m\cdot K$

$I(T) = \int_0^\infty I(\lambda,T)\,d\lambda$; $I = (c/4)u$; $u(\lambda,T) = N(\lambda)E_{av}(\lambda,T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N=\int_0^\infty N(E)\,dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\,dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N=\sum_{n=0}^{\infty} N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^{\infty} E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT}-1}$; $hc = 1{,}240 eV\cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda'-\lambda=\lambda_c(1-\cos\theta)$; $\lambda_c = \dfrac{h}{m_e c} = 0.00243 nm$

double-slit interference maxima: $d\sin\theta = n\lambda$; single-slit diffraction minima: $a\sin\theta = n\lambda$

de Broglie: $\lambda=\dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E=\hbar\omega$; $p=\hbar k$

matter: $E = \dfrac{p^2}{2m}$ (nonrelativistic) or $E = \sqrt{p^2c^2 + m^2c^4}$ (relativistic); photons: $E = pc$

Uncertainty: $\Delta x\Delta k \sim 1$ $\Delta t\Delta\omega \sim 1$; $\Delta x\Delta p \sim \hbar$ $\Delta t\Delta E \sim \hbar$; $\Delta A = \sqrt{<A^2> - <A>^2}$

$\hbar c = 197.3$ eV nm ; group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$

Problem 1 (6 points)
In a Compton scattering experiment, the incoming photon has momentum 100,000 eV/c, and the scattered electron (which was initially at rest) has momentum 100,000 eV/c.
(a) Find the angle at which the photon is scattered relative to the photon's incident direction, in degrees.
(b) Find the angle at which the electron is scattered relative to the photon's incident direction, in degrees.
Use that hc=1240eVnm.

Problem 2 (6 points)
A beam of photons with photon energy 10eV and a beam of electrons with kinetic energy 10eV are incident on a screen that has a narrow slit of width 0.001mm (mm=millimeter). After passing through the slit, photons and electrons are incident on a fluorescent screen that is 1m away, and is parallel to the screen with the slit. A diffraction pattern results for both photons and electrons on the fluorescent screen. Estimate the distance from the first minimum in the diffraction pattern to the center of the diffraction pattern (where the intensity is maximum), for
(a) the photons
(b) the electrons
Give your answers in mm (millimeter) for both cases. Use that hc=1240eVnm.

Problem 3 (6 points)
A non-relativistic electron is described by the wavepacket

$$\psi(x,t) = \int_{k_1}^{k_2} dk \; a(k) e^{i(kx-\omega(k)t)}$$

$$a(k) = A[(k_3)^2 - 7(k-k_0)^2]$$

where A is a constant, and k_0, k_1, k_2, k_3 are constants given by:

$k_0 = 150nm^{-1}, k_1 = 140nm^{-1}, k_2 = 160nm^{-1}, k_3 = 250nm^{-1}$
(a) Estimate the uncertainty in the position of this electron. Give your answer in nm.
(b) Estimate the uncertainty in the momentum of this electron. Give your answer in eV/c.
(c) Estimate the group velocity and the phase velocity of this electron. Give your answers in m/s.
Hint: you don't have to do any integrals to answer these questions.
Justify clearly your answers.

Problem 4 (6 points)
We usually say that the electron mass is $0.511 MeV/c^2$ but in reality we know more: Wikipedia will tell you that we know from experimental measurements that it is $0.51099895000(15) MeV/c^2$, where the numbers in parentheses mean that the last 2 digits (00) could also be anything between +15 and -15. Experiments to date can't tell us more.
(a) What can you deduce about the lifetime of an electron from this information?
(b) Would more accurate experiments in the future be able to get the value of the electron mass to more decimal places or not? Your answer should be either (i)Yes, (ii) No, or (iii) Impossible to tell, and you should justify it.

<u>Problem 5</u> (6 points)
An electron is confined between two impenetrable walls that are 0.001nm apart.
Assuming it is in its lowest possible energy state,
(a) Find its de Broglie wavelength, in nm.
(b) Find its speed v. Give your answer as v/c.