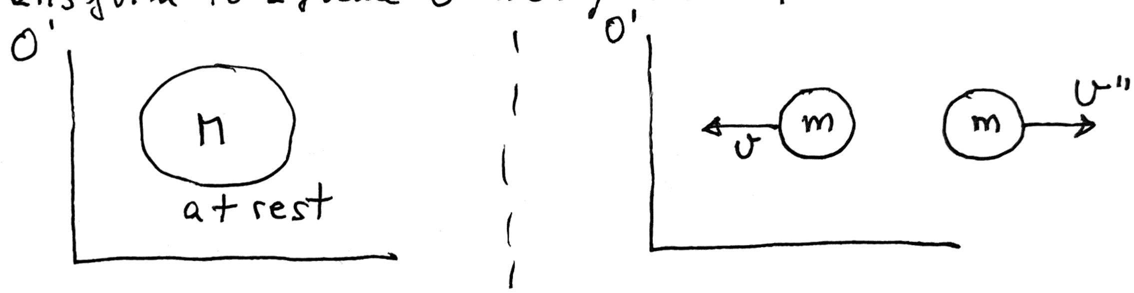
Physics 2D — Quiz 2 Solutions — Fall 2023

Problem 1

(a) classically: $m = M/2$

momentum: $Mv = mv' \Rightarrow \boxed{v' = \frac{M}{m} v = 2v = c}$

(b) Transform to a frame O' moving with speed v



to get v'', we should use the velocity transformation formula:

$$v'' = \frac{v' - v}{1 - v'v/c^2}$$

but in fact, we know that $v'' = v$ because momentum was zero initially and hence it has to be zero at the end also.

By energy conservation we have

$$Mc^2 = 2\gamma(v) mc^2 \Rightarrow \boxed{m = \frac{M}{2\gamma(v)} = 0.433M}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - 0.5^2}} = 1.1547$$

(c) $K_{fin} - K_{in} = \Delta M c^2 = (M - 2 \times 0.433M)c^2 = \boxed{0.134\, Mc^2 = \Delta K}$

Check that results are consistent:

We can find v' from the velocity transformation formula putting $v'' = v$:

$$v = \frac{v' - v}{1 - \frac{v' v}{c^2}} \Rightarrow v - v' \frac{v^2}{c^2} = v' - v \Rightarrow v'\left(1 + \frac{v^2}{c^2}\right) = 2v \Rightarrow$$

$$\Rightarrow \boxed{v' = \frac{2v}{1 + \frac{v^2}{c^2}}} \qquad \gamma(v') = \frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{4v^2/c^2}{\left(1 + \frac{v^2}{c^2}\right)^2}}} =$$

$$= \frac{1}{\sqrt{\frac{\left(1 - \frac{v^2}{c^2}\right)^2}{(1 + v^2/c^2)^2}}} = \boxed{\frac{1 + v^2/c^2}{1 - v^2/c^2} = \gamma(v')}$$

Conservation of momentum in lab frame:

initially: $p_{in} = M \gamma(v) v$ with $\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$p_{final} = m \gamma(v') v' = \frac{M}{\cancel{2}\gamma(v)} \cdot \frac{\cancel{1 + v^2/c^2}}{1 - v^2/c^2} \cdot \frac{\cancel{2}v}{\cancel{1 + v^2/c^2}} = \frac{M}{\sqrt{1 - v^2/c^2}} v = p_{in}$$

Alternative solution to problem 1, without transforming reference frame

momentum: $\gamma(v) M v = \gamma(v') m v'$

energy $\gamma(v) M c^2 = m c^2 + \gamma(v') m c^2$

$\Rightarrow \gamma(v) M = (1+\gamma(v')) m$

from 1st eq.: $m = \dfrac{\gamma(v) M v}{\gamma(v') v'}$. replacing in 2nd eq.

$$\cancel{\gamma(v)}\, \cancel{M} = (1+\gamma(v')) \frac{\cancel{\gamma(v)} \cancel{M} v}{\gamma(v') v'}$$

$\Rightarrow v' \dfrac{\gamma(v')}{1+\gamma(v')} = v$. Now $\gamma(v') = \dfrac{1}{\sqrt{1-v'^2/c^2}} \Rightarrow$

$$\frac{\gamma(v')}{1+\gamma(v')} = \frac{1}{\sqrt{1-v'^2/c^2}} \frac{1}{1+\frac{1}{\sqrt{1-v'^2/c^2}}} = \frac{1}{\sqrt{1-v'^2/c^2}+1} \Rightarrow$$

$$\frac{v'}{\sqrt{1-v'^2/c^2}+1} = v \Rightarrow \frac{\cancel{v'}\left(\sqrt{1-v'^2/c^2}-1\right)}{-\frac{v'^{\cancel{2}}}{c^2}} = v \Rightarrow 1-\sqrt{1-v'^2/c^2} = \frac{v'v}{c^2} \Rightarrow$$

$$\Rightarrow 1-\frac{v'v}{c^2} = \sqrt{1-v'^2/c^2} \Rightarrow 1-\frac{2v'v}{c^2}+\frac{v'^2v^2}{c^4} = 1-\frac{v'^2}{c^2} \Rightarrow$$

$$\Rightarrow \frac{v'^{\cancel{2}}}{\cancel{c^2}}\left(1+\frac{v^2}{c^2}\right) = \frac{2v\cancel{v'}}{\cancel{c^2}} \Rightarrow \boxed{v' = \frac{2v}{1+v^2/c^2}}$$

Next we find $\gamma(v')$: from momentum eq. $\gamma(v') = \dfrac{\gamma(v) M v}{m v'}$. Replacing in energy eq.

$$\gamma(v) M = \left(1+\gamma(v)\frac{Mv}{mv'}\right) m = m + \gamma(v)\frac{v}{v'} M \Rightarrow$$

$$\Rightarrow m = M\gamma(v)\left(1-\frac{v}{v'}\right) = M\gamma(v)\left(1-\frac{\cancel{v}}{2\cancel{v}}\left(1+\frac{v^2}{c^2}\right)\right) = M\gamma(v)\left(\tfrac{1}{2}\right)\underbrace{\left(1-\frac{v^2}{c^2}\right)}_{\frac{1}{\gamma(v)^2}}$$

$$\Rightarrow \boxed{m = \frac{M}{2\gamma(v)}} \boxed{= 0.433M}$$

Problem 2

$p = 0.511\ MeV/c$. Since $m_e c^2 = 0.511\ MeV$, $p = m_e c$

Kinetic energy: $K = (\gamma - 1) mc^2$. Total energy: $E = \gamma mc^2$

Use $E = \sqrt{p^2c^2 + m_e^2c^4} = \sqrt{m_e^2c^4 + m_e^2c^4} = \sqrt{2}\, m_e c^2$

$\Rightarrow \boxed{\gamma = \sqrt{2}}$ $\quad K = (\sqrt{2} - 1) m_e c^2 = 0.414\, m_e c^2$

$\Rightarrow \boxed{K = 0.212\ MeV}$

(b)

$p = \gamma m v, \quad E = \gamma mc^2 \Rightarrow \frac{p}{E} = \frac{v}{c^2} \Rightarrow v = \frac{pc^2}{E} \Rightarrow$

$$v = \frac{m_e c^2 \cdot c}{\gamma m_e c^2} = \frac{c}{\gamma} = \frac{c}{\sqrt{2}} \Rightarrow \boxed{v = 0.707c}$$

Alternative solution:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \Rightarrow$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \Rightarrow \boxed{v = \frac{1}{\sqrt{2}} c}$$

(c) Classically: $p = m_e v$, since $p = m_e c \Rightarrow$

$\boxed{v = c}$

$$K = \frac{1}{2} m_e v^2 = \frac{1}{2} m_e c^2 \Rightarrow \boxed{K = 0.256\ MeV}$$

Problem 3

K_{max} is determined by the smallest wavelength in the range.

let $\lambda_1 = 300\,nm$ $\quad \lambda_2 = 400\,nm$

then $K_1 = \frac{hc}{\lambda_1} - \phi$, $K_2 = \frac{hc}{\lambda_2} - \phi$

first eq: $2K_2 = \frac{hc}{\lambda_1} - \phi$, replacing K_2 from the second eq

$$2\frac{hc}{\lambda_2} - 2\phi = \frac{hc}{\lambda_1} - \phi \Rightarrow \phi = hc\left(\frac{2}{\lambda_2} - \frac{1}{\lambda_1}\right) \Rightarrow$$

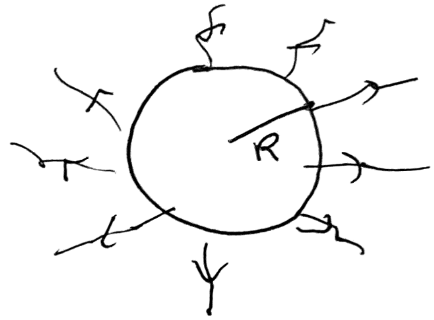
$$\Rightarrow \phi = 1240\,eV\,\cancel{nm}\left(\frac{2}{400\,\cancel{nm}} - \frac{1}{300\,\cancel{nm}}\right) = 2.0666\,eV$$

$\phi = 2.0666\,eV$. The threshold wavelength for that ϕ is given by

$$\phi = \frac{hc}{\lambda_t} \Rightarrow \lambda_t = \frac{hc}{\phi} = \frac{1240\,eV\,nm}{2.0666\,eV} = 600\,nm$$

For incident λ in the range $\boxed{600\,nm \leq \lambda \leq \infty}$ there will be no photoelectrons emitted

Problem 4



(a) $\lambda_m T = 2.8978 \times 10^{-3}\, m\, K$. $\lambda_m = 1000\, nm \Rightarrow$

$$T = \frac{2.8978 \times 10^{-3}\, m\, K}{10^3 \times 10^{-9}\, m} = 2.8978 \times 10^3\, K \Rightarrow$$

$$\boxed{T = 2{,}898\, K}$$

(b) Power emitted is $P = \sigma T^4 . A$, where A is surface area of sphere. $A = 4\pi R^2 \Rightarrow$

$$P = \sigma T^4 . 4\pi R^2 \Rightarrow R^2 = \frac{P}{\sigma T^4 . 4\pi} \Rightarrow$$

$$R = \sqrt{\frac{P}{4\pi \sigma T^4}} = \sqrt{\frac{5000 W \quad m^2 K^4}{4\pi . 5.67037 \times 10^{-8} W . 2898^4 K^4}}$$

$$\Rightarrow R = 0.00997\, m \Rightarrow \boxed{R = 1 cm}$$

(c) The temperature at the center of the sphere can be anything, we have no information about it.

Problem 5

$$I(\lambda) = \frac{8\pi}{\lambda^4}\frac{hc}{\lambda}\frac{1}{e^{hc/\lambda kT}-1}\cdot\frac{c}{4}$$

emits maximum power at $\lambda_m = 100\,nm$.

We know that $\frac{hc}{\lambda_m kT} = 4.96$

$$\Rightarrow I(\lambda_m) = \frac{8\pi hc}{\lambda_m^5}\frac{1}{e^{4.96}-1}\frac{c}{4}$$

At $\lambda = 200\,nm = 2\lambda_m$: $\frac{hc}{\lambda kT} = \frac{hc}{2\lambda_m kT} = \frac{4.96}{2} = 2.48$

$$I(\lambda) = I(2\lambda_m) = \frac{8\pi hc}{(2\lambda_m)^5}\frac{1}{e^{2.48}-1}\frac{c}{4}$$

$$\Rightarrow I(2\lambda_m) = \frac{1}{2^5}\frac{8\pi hc}{\lambda_m^5}\frac{1}{e^{4.96}-1}\frac{e^{4.96}-1}{e^{2.48}-1}\frac{c}{4} \Rightarrow$$

$$I(2\lambda_m) = \frac{1}{2^5}\frac{e^{4.96}-1}{e^{2.48}-1}I(\lambda_m) = \frac{1}{32}\times 12.94\times I(\lambda_m)$$

since $I(\lambda_m) = 1000\,W/nm$

$$\boxed{I(2\lambda_m) = \frac{12.94}{32}\times 1000\,\frac{W}{nm} = 404\,\frac{W}{nm}}$$

(b) At wavelength λ_m and twice the temperature

$$\frac{hc}{\lambda_m k.(2T)} = \frac{1}{2}\frac{hc}{\lambda_m kT} = 2.48$$

$$\Rightarrow I(\lambda_m, 2T) = \frac{8\pi hc}{\lambda_m^5}\frac{1}{e^{2.48}-1}\frac{c}{4} = I(\lambda_m, T)\cdot\frac{e^{4.96}-1}{e^{2.48}-1}$$

$$\Rightarrow I(\lambda_m, 2T) = 12.94\times I(\lambda_m, T) = \boxed{12{,}940\,\frac{W}{nm}}$$