Physics 2D — Quiz 1 Solutions — Fall 2023

Problem 1

light travels at speed c relative to the ether.

So on earth, light travels to the right at speed $v_r = c - u$,

and to the left at speed $v_l = c + u$.

(a) Light reaches screen on the left first, since it travels faster in that direction.

(b.) $$\Delta t_{right} = \frac{L}{c-u}, \quad \Delta t_{left} = \frac{L}{c+u}$$

$$\Delta t = \Delta t_{right} - \Delta t_{left} = L\left(\frac{1}{c-u} - \frac{1}{c+u}\right) = \frac{Lu \cdot 2}{c^2 - u^2}$$

$$\Rightarrow \Delta t = \frac{2Lu}{c^2} \cdot \frac{1}{1 - u^2/c^2}$$ since $\frac{u}{c} \ll 1$, $\frac{u^2}{c^2} \ll 1$ and we can ignore it.

$$\Rightarrow \Delta t = \frac{2Lu}{c^2} \Rightarrow u = \frac{c^2}{2L} \cdot \Delta t$$

$$\Rightarrow u = \frac{(3 \times 10^8)^2 \, m^{\cancel{2}}/s^{\cancel{2}}}{150 \cancel{m} \cdot 2} \cdot 10^{-14} \cancel{s} = \frac{9 \times 10^{16}}{300} \times 10^{-14} \frac{m}{s} = \frac{900}{300} \frac{m}{s}$$

$$\Rightarrow \boxed{u = 3 \, m/s}$$

Problem 2

The distance between earth and Andromeda is

$$D = c \cdot 9\,\text{years}$$

The time that passes in the traveler's reference frame is the proper time $\Delta t_0 = \frac{\Delta t}{\gamma}$, $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$

For traveling at speed u, the time it takes ~~one~~ to travel distance D as measured in the earth's reference frame is

$$\Delta t = \frac{D}{u}$$

$$\Rightarrow \quad \Delta t_0 = \frac{D}{u\gamma} \Rightarrow u\gamma = \frac{D}{\Delta t_0}$$

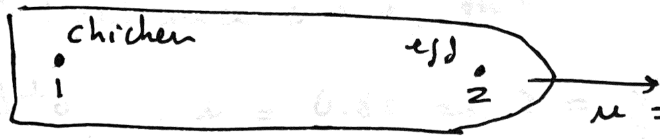
We want $\Delta t_0 = 9\ \text{months} = \frac{9}{12}\ \text{years}$

$$\Rightarrow \quad \frac{D}{\Delta t_0} = \frac{c \cdot 9\,\text{years}}{\frac{9}{12}\,\text{years}} = 12c$$

$$\Rightarrow \quad \frac{u}{c}\gamma = 12 \Rightarrow \frac{u}{c}\frac{1}{\sqrt{1-u^2/c^2}} = 12 \Rightarrow \frac{\frac{u^2}{c^2}}{1-u^2/c^2} = 144$$

$$\Rightarrow \quad 145\frac{u^2}{c^2} = 144 \Rightarrow \boxed{\frac{u}{c} = \sqrt{\frac{144}{145}} = 0.9965}$$

Problem 3



$u = 0.8c = \frac{4}{5}c$

event 1 : chicken is born

event 2 : hen lays egg

$$\gamma = \frac{1}{\sqrt{1-\frac{16}{25}}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3}$$

$$t_1 = \gamma\left(t_1' + \frac{u x_1'}{c^2}\right)$$

$$t_2 = \gamma\left(t_2' + \frac{u x_2'}{c^2}\right)$$

$$t_2 - t_1 = \gamma(t_2' - t_1') + \gamma\frac{u}{c^2}(x_2' - x_1')$$

$$x_2' - x_1' = L_0 = 300\,\text{m}$$

$$t_2 - t_1 = x\ \mu s \quad ; \quad t_2' - t_1' = -x\ \mu s = -(t_2 - t_1)$$

$$\Rightarrow (1+\gamma)(t_2 - t_1) = \gamma\frac{u}{c}\frac{L_0}{c} \quad \Rightarrow$$

$$t_2 - t_1 = \frac{\gamma}{1+\gamma}\frac{u}{c}\frac{L_0}{c} = \frac{\cancel{5}/\cancel{3}}{\frac{8}{\cancel{3}}}\cdot\frac{4}{\cancel{5}}\cdot\frac{\overset{100}{\cancel{300\,\text{m}}}}{\cancel{3}\cdot 10^8\,\cancel{\text{m}}}\cdot\text{s} = \frac{4}{8}\cdot 10^{-6}\,\text{s}$$

$$\Rightarrow \boxed{x = 0.5\ \mu s}$$

Problem 4

(a) In the earth's reference frame, twin B lights candle at time $\Delta t = \gamma \Delta t_0$. $u = 0.8c \Rightarrow \gamma = \frac{5}{3}$

$\Delta t_0 = 1$ year $\Rightarrow$ $\Delta t = \frac{5}{3}$ years.

The distance traveled by twin B during time Δt is:

$$D = u \Delta t = \frac{4}{5} c \cdot \frac{5}{3} \text{ years} = \frac{4}{3} c \cdot \text{year}$$

The time it takes the light emitted by twin B's candle to reach the earth from the moment it is emitted is:

$$\Delta t_2 = \frac{D}{c} = \frac{4}{3} \text{ year}$$

$\Rightarrow$ the light reaches twin A after total time

$$\Delta t_{total} = \Delta t + \Delta t_2 = \frac{5}{3} \text{ years} + \frac{4}{3} \text{ years} = 3 \text{ years}$$

So $\boxed{\text{A is exactly 23 years old when light reaches A.}}$

(b) The situation is symmetric.

Problem 5

(a) red light looks blue to me $\Rightarrow$ police car is approaching me $\Rightarrow$ $u > 0.5c$ $\Rightarrow$ $\boxed{\text{police car will catch up.}}$

(b) $f' = f\sqrt{\dfrac{1+v/c}{1-v/c}}$. Here, v = speed of police car relative to me, and f' = blue light frequency, f = red light frequency

$$\Rightarrow \frac{f'}{f} = \sqrt{\frac{1+v/c}{1-v/c}} \Rightarrow \left(\frac{f'}{f}\right)^2 = \frac{1+v/c}{1-v/c} \Rightarrow \frac{v}{c}\left(1+\left(\frac{f'}{f}\right)^2\right) = \left(\frac{f'}{f}\right)^2 - 1$$

$$\Rightarrow \frac{v}{c} = \frac{(f'/f)^2 - 1}{1+(f'/f)^2} \quad ; \quad \frac{f'}{f} = \frac{\lambda_{red}}{\lambda_{blue}} = \frac{7}{5} \Rightarrow$$

$$\frac{v}{c} = \frac{(7/5)^2 - 1}{1+(7/5)^2} = \frac{24}{74} = 0.324 \Rightarrow$$ $\boxed{\text{police car is approaching me at speed } 0.324c}$

(c) Put reference frame O' on police car.

$$v'_x = \frac{v_x - u}{1 - u v_x/c^2} \qquad v_x = 0.5c, \quad v'_x = -0.324c$$

solve for u: $v'_x - \dfrac{u v'_x v_x}{c^2} = v_x - u \Rightarrow u\left(1 - \dfrac{v_x v'_x}{c^2}\right) = v_x - v'_x \Rightarrow$

$$\Rightarrow u = \frac{v_x - v'_x}{1 - \frac{v_x v'_x}{c^2}} = \frac{0.5c + 0.324c}{1 + 0.5 \times 0.324} = 0.709c \qquad \boxed{u = 0.709c}$$