

**PHYSICS 140B : STATISTICAL PHYSICS  
MIDTERM EXAMINATION**

(1) A one-dimensional spin chain is described by the Hamiltonian

$$\hat{H} = - \sum_n \left( JS_n S_{n+1} + K S_n^2 S_{n+1}^2 \right) ,$$

where  $S_n \in \{-1, 0, +1\}$  on each site. Find the transfer matrix. [50 points]

(2) Consider a gas of ballistic particles in  $d = 2$  dimensions with two-body interactions

$$u(r) = \frac{A}{r^3} .$$

(a) For the cluster  $\gamma = \bullet\text{---}\bullet$ , express  $b_\gamma$  as an integral over the radial coordinate and show that the expression is integrable both as  $r \rightarrow 0$  and as  $r \rightarrow \infty$ . [20 points]

(b) Compute the second virial coefficient  $B_2(T)$ . Express any integral expressions as dimensionless integrals with dimensionful prefactors. You may find the following useful:

$$\int_0^\infty ds \frac{1 - e^{-s}}{s^{5/3}} = -\Gamma(-\frac{2}{3}) = 4.01841 \dots .$$

[30 points]