PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) For the Mayer cluster expansion, write down all possible unlabeled connected subgraphs γ which contain four vertices. For your favorite of these animals, identify its symmetry factor s_γ, and write down the corresponding expression for the cluster integral b_γ. For example, for the $\Box$ diagram with four vertices the symmetry factor is $s_\Box = 8$ and the cluster integral is

$$\begin{aligned} b_\Box &= \frac{1}{8V}\int\! d^d\!r_1 \int\! d^d\!r_2 \int\! d^d\!r_3 \int\! d^d\!r_4\, f(r_{12})\, f(r_{23})\, f(r_{34})\, f(r_{14}) \\ &= \frac{1}{8}\int\! d^d\!r_1 \int\! d^d\!r_2 \int\! d^d\!r_3\, f(r_{12})\, f(r_{23})\, f(r_1)\, f(r_3) \quad . \end{aligned}$$

(You'll have to choose a favorite other than $\Box$.) If you're really energetic, compute s_γ and b_γ for all of the animals with four vertices.

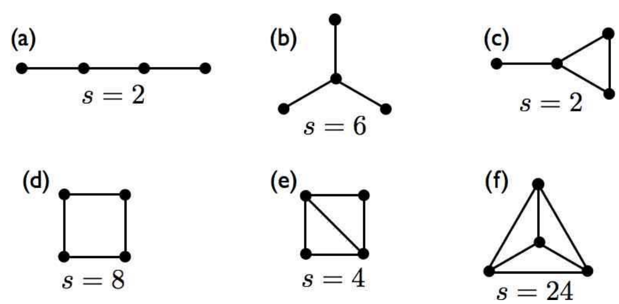

Figure 1: Connected clusters with $n_\gamma = 4$ sites.

(2) For each of the cluster diagrams in Fig. 2, find the symmetry factor s_γ and write an expression for the cluster integral b_γ.

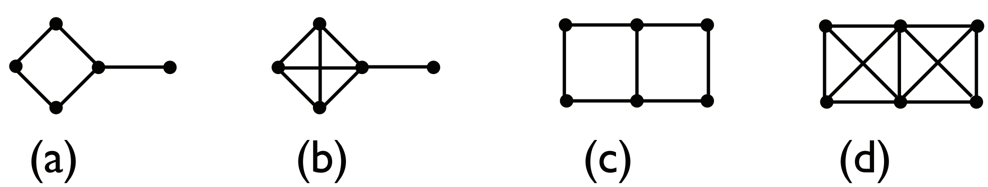

Figure 2: Cluster diagrams for problem 2.

(3) Compute the partition function for the one-dimensional Tonks gas of hard rods of length a on a ring of circumference L. This is slightly tricky, so here are some hints. Once again, assume a particular ordering so that $x_1 < x_2 < \cdots < x_N$. Due to translational invariance, we can define the positions of particles $\{2, \ldots, N\}$ relative to that of particle 1, which we initially place at $x_1 = 0$. Then periodicity means that $x_N \le L - a$, and in general one then has

$$x_{j-1} + a \le x_j \le Y_j \equiv L - Na + (j-1)a \quad .$$

Now integrate over $\{x_2, \ldots, x_N\}$ subject to these constraints. Finally, one does the x_1 integral, which is over the entire ring, but which must be corrected to eliminate overcounting from cyclic permutations. How many cyclic permutations are there?

(4) Consider a three-dimensional gas of point particles interacting according to the potential

$$u(r) = \begin{cases} +\Delta_0 & \text{if } r \leq a \\ -\Delta_1 & \text{if } a < r \leq b \\ 0 & \text{if } b < r \quad , \end{cases}$$

where $\Delta_{0,1}$ are both positive. Compute the second virial coefficient $B_2(T)$ and find a relation which determines the inversion temperature in a throttling process.