PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #1

(1) Consider a two-dimensional gas of fermions which obey the dispersion relation

$$\varepsilon(\boldsymbol{k}) = \varepsilon_0\Big((k_x^2+k_y^2)\,a^2 + \tfrac{1}{2}(k_x^4+k_y^4)\,a^4\Big) \quad .$$

Sketch, on the same plot, the Fermi surfaces for $\varepsilon_{\mathrm{F}} = 0.1\,\varepsilon_0$, $\varepsilon_{\mathrm{F}} = \varepsilon_0$, and $\varepsilon_{\mathrm{F}} = 10\,\varepsilon_0$.

(2) Using the Sommerfeld expansion, compute the heat capacity for a two-dimensional electron gas, to lowest nontrivial order in the temperature T.

(3) ^{3}He atoms consist of an odd number of fermions (two electrons, two protons, and one neutron), and hence is itself a fermion. Consider a kilomole of ^{3}He atoms at standard temperature and pressure ($T = 293,\ \mathrm{K}$, $p = 1\,\mathrm{atm}$).

(a) What is the Fermi temperature of the gas?

(b) Calculate $\mu/k_{\mathrm{B}}T$ and $\exp(-\mu/k_{\mathrm{B}}T)$.

(c) Find the average occupancy $n(\varepsilon)$ of a single particle state with energy $\frac{3}{2}k_{\mathrm{B}}T$.

(4) For ideal Fermi gases in $d = 1$, 2, and 3 dimensions, compute at $T = 0$ the average energy per particle E/N in terms of the Fermi energy ε_{F}.

(5) Obtain numerical estimates for the Fermi energy (in eV) and the Fermi temperature (in Kelvin) for the following systems:

(a) conduction electrons in silver, lead, and aluminum

(b) nucleons in a heavy nucleus, such as $^{200}\mathrm{Hg}$

(6) Show that the chemical potential of a three-dimensional ideal nonrelativistic Fermi gas is given by

$$\mu(n,T) = \varepsilon_{\mathrm{F}}\left[1 - \frac{\pi^2}{12}\left(\frac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}}\right)^2 - \frac{\pi^4}{80}\left(\frac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}}\right)^4 + \ldots\right]$$

and the average energy per particle is

$$\frac{E}{N} = \tfrac{3}{5}\,\varepsilon_{\mathrm{F}}\left[1 + \frac{5\pi^2}{12}\left(\frac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}}\right)^2 - \frac{\pi^4}{16}\left(\frac{k_{\mathrm{B}}T}{\varepsilon_{\mathrm{F}}}\right)^4 + \ldots\right] \quad ,$$

where $\mu_0(n)$ is the Fermi energy at $T = 0$. Compute the heat capacity $C_V(T)$ to terms of order T^3. How does the T^3 contribution to the electronic heat capacity compare with the contribution from phonons?