PHYSICS 140B : STATISTICAL PHYSICS
FINAL EXAM

(1) Consider the equation of state

$$p = \frac{RT}{v - b\sqrt{T}} \exp\bigg(-\frac{a}{v\sqrt{T}}\bigg) \quad ,$$

where a and b are constants, $R = N_{\rm A} k_{\rm B}$ is the gas constant, and v is the molar volume.

(a) Find the critical point values of $p_{\rm c}$, $T_{\rm x}$, and $v_{\rm c}$. [12 points]

(b) Find the dimensionless equation of state $\bar{p} = \bar{p}(\bar{T}, \bar{v})$, where $\bar{p} = p/p_{\rm c}$, $\bar{T} = T/T_{\rm c}$, and $\bar{v} = v_{\rm c}$. Check that $\bar{p}(1,1) = 1$. [11 points]

(c) Describe the difference between the coexistence boundary and the spinodal line in a (p, v) diagram. Illustrate with a sketch, including curves for $p(T, v)$ with $T > T_{\rm c}$, $T = T_{\rm c}$, and $T < T_{\rm c}$. [10 points]

(2) The Hamiltonian for the four state clock model can be written as

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{\boldsymbol{n}}_i \cdot \hat{\boldsymbol{n}}_j - \boldsymbol{H} \cdot \sum_i \hat{\boldsymbol{n}}_i \quad ,$$

where on each site i the local 'spin' takes one of four possible vector values: $\hat{\boldsymbol{n}}_i \in \{\pm\hat{\boldsymbol{x}}, \pm\hat{\boldsymbol{y}}\}$. The interactions are between nearest neighbors on a lattice of coordination number z. The applied field is $\boldsymbol{H} = H\hat{\boldsymbol{x}}$.

(a) Make the mean field *Ansatz* $\hat{\boldsymbol{n}}_i = \boldsymbol{m} + \delta\hat{\boldsymbol{n}}_i$, with $\boldsymbol{m} = m\hat{\boldsymbol{x}}$. Find the mean field Hamiltonian $\hat{H}^{\rm MF}$. [7 points]

(b) Defining $\theta \equiv k_{\rm B}T/zJ$ and $h \equiv H/zJ$, find the dimensionless mean field free energy per site $f \equiv F/NzJ$ as a function of θ, h, and m. [7 points]

(c) What is the self-consistent equation for m? [7 points]

(d) For $h = 0$, what is the critical temperature $\theta_{\rm c}$? Is the transition first order (discontinuous) or second order (continuous)? Why? [7 points]

(e) For $\theta > \theta_{\rm c}$, find $m(\theta, h)$ to first order in h. [6 points]

(3) Consider an ideal gas of particles obeying the dispersion $\varepsilon(\boldsymbol{p}) = Ap^{3/2}$ in $d = 3$ dimensions.

(a) Find the scattering time $\tau(T)$. You are only asked to find the functional dependence on T, including the proper combination of dimensional prefactors involving A, $k_{\rm B}$, n (number density), and σ (scattering cross section). You do not need to compute any numerical coefficients. *Hint: First find $\bar{v}_{\rm rel}(T)$ on dimensional grounds. Then use this to obtain $\tau(T)$.* [15 points]

(b) Find the thermal conductivity $\kappa(T)$ from the relation

$$\kappa = \frac{n\tau}{3k_{\rm B}T^2} \left\langle \boldsymbol{v}^2 \varepsilon(\varepsilon - c_p T) \right\rangle \quad .$$

Once again, you are asked only to find the correct temperature dependence and combination of dimensional prefactors – don't bother with the numerical prefactors. *Hint : This means you can solve this problem using dimensional analysis.* `[18 points]`