PHYSICS 210A : EQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #6

(1) For the Mayer cluster expansion, write down all possible unlabeled connected subgraphs γ which contain four vertices. For your favorite of these animals, identify its symmetry factor s_γ, and write down the corresponding expression for the cluster integral b_γ. For example, for the $\Box$ diagram with four vertices the symmetry factor is $s_\Box = 8$ and the cluster integral is

$$\begin{aligned} b_\Box &= \frac{1}{8V} \int\! d^d\!r_1 \int\! d^d\!r_2 \int\! d^d\!r_3 \int\! d^d\!r_4 \, f(r_{12})\, f(r_{23})\, f(r_{34})\, f(r_{14}) \\ &= \frac{1}{8} \int\! d^d\!r_1 \int\! d^d\!r_2 \int\! d^d\!r_3 \, f(r_{12})\, f(r_{23})\, f(r_1)\, f(r_3) \quad . \end{aligned}$$

(You'll have to choose a favorite other than $\Box$.) If you're really energetic, compute s_γ and b_γ for all of the animals with four vertices.

(2) Consider a three-dimensional gas of point particles interacting according to the potential

$$u(r) = \begin{cases} +\Delta_0 & \text{if } r \le a \\ -\Delta_1 & \text{if } a < r \le b \\ 0 & \text{if } b < r \quad , \end{cases}$$

where $\Delta_{0,1}$ are both positive. Compute the second virial coefficient $B_2(T)$ and find a relation which determines the inversion temperature in a throttling process.

(3) Consider a liquid where the interaction potential is $u(r) = \Delta_0\,(a/r)^k$, where Δ_0 and a are energy and length scales, respectively. Assume that the pair distribution function is given by $g(r) \approx e^{-u(r)/k_B T}$. Compute the equation of state. For what values of k do your expressions converge?

(4) Consider the equation of state

$$p\sqrt{v^2 - b^2} = RT\, \exp\!\left(-\frac{a}{RTv^2}\right) .$$

(a) Find the critical point (v_c, T_c, p_c).

(b) Defining $\bar{p} = p/p_c$, $\bar{v} = v/v_c$, and $\bar{T} = T/T_c$, write the equation of state in dimensionless form $\bar{p} = \bar{p}(\bar{v}, \bar{T})$.

(c) Expanding $\bar{p} = 1 + \pi$, $\bar{v} = 1 + \epsilon$, and $\bar{T} = 1 + t$, find $\epsilon_{liq}(t)$ and $\epsilon_{gas}(t)$ for $-1 \ll t < 0$.