PHYSICS 210A : EQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #4

(1) Consider a three dimensional gas of particles with dispersion $\varepsilon(\boldsymbol{k}) = \varepsilon_0 (ka)^{3/2}$, where ε_0 and a are microscopic energy and length scales, respectively.

(a) Find the density of states per unit volume $g(\varepsilon)$. You may assume there are no internal degeneracies.

(b) Find an expression for the expansion coefficients $C_j(T)$ defined in eqn. 5.33 of the lecture notes.

(c) Find the virial coefficients $B_j(T)$ up through $j = 5$. It is convenient to use the `Mathematica` function `InverseSeries`. For guidance, see example problem 5.13.

(2) In our derivation of the low temperature phase of an ideal Bose condensate, we split off the lowest energy state ε_0 but treated the remainder as a continuum, taking $\mu = 0$ in all expressions relating to the overcondensate. Under what conditions is this justified? *I.e.* why are we not obligated to separately consider the contributions from the first excited state, *etc.*?

(3) Consider a three-dimensional Bose gas of particles which have two internal polarization states, labeled by $\sigma = \pm 1$. The single particle energies are given by

$$\varepsilon(\boldsymbol{p}, \sigma) = \frac{\boldsymbol{p}^2}{2m} + \sigma\Delta \ ,$$

(a) Find the density of states per unit volume $g(\varepsilon)$.

(b) Find an implicit expression for the condensation temperature $T_c(n, \Delta)$. When $\Delta \to \infty$, your expression should reduce to the familiar one derived in class.

(c) When $\Delta = \infty$, the condensation temperature should agree with the familiar result for three-dimensional Bose condensation. Assuming $\Delta \gg k_B T_c(n, \Delta = \infty)$, find analytically the leading order difference $T_c(n, \Delta) - T_c(n, \Delta = \infty)$.

(4) A branch of excitations for a three-dimensional system has a dispersion $\varepsilon(\boldsymbol{k}) = A\,|\boldsymbol{k}|^{2/3}$. The excitations are bosonic and are not conserved; they therefore obey photon statistics.

(a) Find the single excitation density of states per unit volume, $g(\varepsilon)$. You may assume that there is no internal degeneracy for this excitation branch.

(b) Find the heat capacity $C_V(T, V)$.

(c) Find the ratio E/pV.

(d) If the particles are bosons with number conservation, find the critical temperature T_c for Bose-Einstein condensation.