PHYSICS 210A : EQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #3

(1) Consider a system composed of spin tetramers, each of which is described by the Hamiltonian

$$\hat{H} = -J(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_1\sigma_4 + \sigma_2\sigma_3 + \sigma_2\sigma_4 + \sigma_3\sigma_4) - \mu_0 H(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \quad .$$

The individual tetramers are otherwise noninteracting.

(a) Find the single tetramer partition function ζ.

(b) Find the magnetization per tetramer $m = \mu_0 \langle \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 \rangle$.

(c) Suppose the tetramer number density is n_{t}. The magnetization density is $M = n_{\mathrm{t}} m$. Find the zero field susceptibility $\chi(T) = (\partial M/\partial H)_{H=0}$.

(2) A surface consisting of N_{s} adsorption sites is in thermal and particle equilibrium with an ideal monatomic gas. Each adsorption site can accommodate either zero particles (energy 0), one particle (two states, each with energy ε), or two particles (energy $2\varepsilon + U$).

(a) Find the grant partition function of the surface, $\Xi_{\mathrm{surf}}(T, N_{\mathrm{s}}, \mu)$. and the surface grand potential $\Omega_{\mathrm{surf}}(T, N_{\mathrm{s}}, \mu)$.

(b) Find the fraction of adsorption sites with are empty, singly occupied, and double occupied. Express your answer in terms of the temperature, the density of the gas, and other constants.

(3) Consider the Hamiltonian below with $\mathcal{N}$ kinetic DOF, $\mathcal{N}_2$ quadratic potential DOF, and $\mathcal{N}_4$ quartic DOF, with $\mathcal{N} \geq \mathcal{N}_2 + \mathcal{N}_4$.

$$\hat{H} = \tfrac{1}{2} \sum_{i,j}^{\mathcal{N}} m_{ij}^{-1} \, p_i \, p_j + \tfrac{1}{2} \sum_{i=1}^{\mathcal{N}_2} K_i \, q_i^2 + \tfrac{1}{4} \sum_{j=\mathcal{N}_2+1}^{\mathcal{N}_2+\mathcal{N}_4} A_j \, q_j^4 \quad .$$

Find the free energy F and the internal energy E in terms of T, V, $\mathcal{N}$, $\mathcal{N}_2$, and $\mathcal{N}_4$. Assume the matrix m_{ij} is nondegenerate.

(4) Consider a gas of classical spin-$\frac{3}{2}$ particles, with Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m} - \mu_0 H \sum_i S_i^z \quad ,$$

where $S_i^z \in \big\{ -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2} \big\}$ and H is the external magnetic field. Find the Helmholtz free energy $F(T, V, H, N)$, the entropy $S(T, V, H, N)$, and the magnetic susceptibility $\chi(T, H, n)$, where $n = N/V$ is the number density.