Physics 152B/232 Spring 2025

Problem Set 4

Problem 1

Consider a two-site generalized Hubbard model

$$H = -t\sum_{\sigma}(c^{+}_{1\sigma}c_{2\sigma} + h.c.) + U(n_{1\uparrow}n_{1\downarrow} + n_{2\uparrow}n_{2\downarrow}) + X\sum_{\sigma}(c^{+}_{1\sigma}c_{2\sigma} + h.c.)(n_{1,-\sigma} + n_{2,-\sigma})$$

with t>0 and X>0 and U>0. Assume X<t.

(a) Repeat what you did in Problem 3 of HW Set 3 and find expressions for $U_{eff}(n=0)$ and $U_{eff}(n=2)$ in terms of t, U and X.

(b) Find conditions on the parameters such that U_{eff} <0, i.e. the effective interaction is attractive. Can that happen for n=0? For n=2?

Problem 2

Consider the reduced Hamiltonian

$$H = \sum_{k\sigma}\varepsilon_k c^{+}_{k\sigma}c_{k\sigma} + \sum_{kk'}V_{kk'}c^{+}_{k\uparrow}c^{+}_{-k\downarrow}c_{-k'\downarrow}c_{k'\uparrow}$$

that results from the Hamiltonian considered in Problem 5 of HW3 with t_{ij} and U and X_{ij} and $\alpha = X_{ij}/t_{ij} > 0$ and $t_{00} = 0$, and assume $U > 0$.

(a) Find the equation that needs to be solved to obtain the binding energy of a Cooper pair with this Hamiltonian.

(b) Show that if $\alpha = 0$ no Cooper pair can form.

(c) Discuss qualitatively whether or not Cooper pairs may form if $\alpha > 0$ when the band is close to empty and when it is close to full.

(d) Assuming the system is a one-dimensional chain with N=6 sites and nearest neighbor hopping only, and the Fermi sea has 6 electrons, find examples of parameters that give rise to bound Cooper pairs.

Problem 3

Consider the tight binding energy band for a two-dimensional square lattice with lattice spacing a and nearest neighbor hopping t. Calculate the electrical conductivity from the Boltzmann equation within the relaxation time approximation for:

(a) the band nearly empty

(b) the band half-filled

(c) Writing the conductivities in the form given by the Drude formula in terms of a transport effective mass, what is the ratio of transport effective masses for the two cases?

Problem 4

Consider a body of volume V with simple cubic Bravais lattice structure , with conduction electrons in an energy band with energy versus k relation ε_k . Define

$$\frac{1}{m_k^*} = \frac{1}{\hbar^2}\frac{\partial^2 \varepsilon_k}{\partial k^2}$$

(a) Assuming the body has zero electrical resistance, show that the current density $\vec{J}$ that develops when a magnetic field $\vec{H}$ is applied satisfies the London equation

$$\vec{\nabla} \times \vec{j} = -\frac{1}{\mu_0 \lambda_L^2}\vec{B}$$

and find an expression for the London penetration depth λ_L in terms of m_k^*'s.

(b) Discuss the behavior of λ_L as function of the occupation of the band n and make a qualitative plot of λ_L versus n for $0 \le n \le 2$.

(c) For the two-dimensional square lattice described by a tight binding energy band with nearest neighbor hopping t=0.25eV, and lattice spacing a=1A, find the numerical value of λ_L in Angstroms when the band is half-filled.

Problem 5

Lithium has electronic configuration $1s^2 2s^1$. It crystalizes in a bcc structure with lattice constant a=3.49A.

(a) Assume you have a crystal composed of Li^+ ions in the same crystal configuration. Estimate its magnetic susceptibility. Is it paramagnetic or diamagnetic?
(b) Same as (a) for a crystal of Li atoms, not ions.