PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #6

(1) Consider a two-dimensional gas of identical classical, noninteracting, massive relativistic particles with dispersion $\varepsilon(\boldsymbol{p}) = \sqrt{\boldsymbol{p}^2 c^2 + m^2 c^4}$.

(a) Compute the free energy $F(T, V, N)$.

(b) Find the entropy $S(T, V, N)$.

(c) Find an equation of state relating the fugacity $z = e^{\mu/k_{\rm B}T}$ to the temperature T and the pressure p.

(2) A box of volume V contains N_1 identical atoms of mass m_1 and N_2 identical atoms of mass m_2.

(a) Compute the density of states $D(E, V, N_1, N_2)$.

(b) Let $x_1 \equiv N_1/N$ be the fraction of particles of species #1. Compute the statistical entropy $S(E, V, N, x_1)$.

(c) Under what conditions does increasing the fraction x_1 result in an increase in statistical entropy of the system? Why?

(3) Consider a monatomic gas of N identical particles of mass m in three space dimensions. The Hamiltonian of each particle is

$$\hat{h} = \frac{\boldsymbol{p}^2}{2m} + \hat{h}_{\rm el} \quad ,$$

where $\hat{h}_{\rm el}$ is an electronic Hamiltonian with $(g+1)$ levels: a nondegenerate ground state at energy $\varepsilon_0 = 0$ and a g-fold degenerate excited state at energy $\varepsilon_1 = \Delta$.

(a) What is the single particle partition function ζ. Assume the system is confined to a box of volume V.

(b) What is the Helmholtz free energy $F(T, V, N)$?

(c) What is the heat capacity at constant volume $C_V(T, V, N)$? Interpret your result.

(4) A surface consisting of $N_{\rm s}$ adsorption sites is in thermal and particle equilibrium with an ideal monatomic gas. Each adsorption site can accommodate either zero particles (energy 0), one particle (two states, each with energy ε), or two particles (energy $2\varepsilon + U$).

(a) Find the grant partition function of the surface, $\Xi_{\rm surf}(T, N_{\rm s}, \mu)$. and the surface grand potential $\Omega_{\rm surf}(T, N_{\rm s}, \mu)$.

(b) Find the fraction of adsorption sites with are empty, singly occupied, and double occupied. Express your answer in terms of the temperature, the density of the gas, and other constants.

(5) A classical gas of indistinguishable particles in three dimensions is described by the Hamiltonian

$$\hat{H} = \sum_{i=1}^{N} \Big\{ A\,|\mathbf{p}_i|^3 - \mu_0 H S_i \Big\} \quad ,$$

where A is a constant, and where $S_i \in \{-1\,,\,0\,,\,+1\}$ (*i.e.* there are three possible spin polarization states).

(a) Compute the free energy $F_{\textsf{gas}}(T,H,V,N)$.

(b) Compute the magnetization density $m_{\textsf{gas}} = M_{\textsf{gas}}/V$ as a function of temperature, pressure, and magnetic field.

The gas is placed in thermal contact with a surface containing $N_{\textsf{s}}$ adsorption sites, each with adsorption energy $-\Delta$. The surface is metallic and shields the adsorbed particles from the magnetic field, so the field at the surface may be approximated by $H=0$.

(c) Find the Landau free energy for the surface, $\Omega_{\textsf{surf}}(T,N_{\textsf{S}},\mu)$.

(d) Find the fraction $f_0(T,\mu)$ of empty adsorption sites.

(e) Find the gas pressure $p^*(T,H)$ at which $f_0 = \frac{1}{2}$.