PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) A substance obeys the thermodynamic relation $E = aS^4/VN^2$.

(a) Compute the heat capacity $C_{V,N}$ in terms of N, V, and T.

(b) Compute the equation of state relating p, V, N, and T.

(c) Compute the ratio $C_{\varphi,N}/C_{V,N}$, where $C_{\varphi,N}$ is the heat capacity at constant φ and N, with $\varphi = V^2/T$.

(2) A strange material satisfies $E(S,V,N) = a\,S^6/V^3N^2$.

(a) What are the SI dimensions of a?

(b) Find the equation of state relating p, T, and $n = N/V$.

(c) Find the coefficient of thermal expansion $\alpha_{\rm p} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$. Express your answer in terms of T.

(d) Find the coefficient of isothermal compressibility $\kappa_{\rm T} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$. Express your answer in terms of p.

(3) ν moles of the substance in problem 2 execute a Carnot cycle between reservoirs at temperatures T_1 and T_2. The top isotherm extends from volume $V_{\rm A}$ to $V_{\rm B}$. Find the heat Q and work W for each leg of the cycle, and compute the cycle efficiency.
Suggestion: It is useful to use §2.6.6 of the Lecture Notes as a template.

(4) An interacting diatomic gas obeys the equation of state

$$p(v-b) = RT\,e^{-a/v}\ ,$$

where $v = N_{\rm A}V/N$ is the molar volume.

(a) Show that $E(T,V,N) = \frac{1}{2}fNk_{\rm B}T$, the same as for an ideal gas.

(b) Find the molar specific heat c_p as a function of the specific volume v.