PHYSICS 200B : CLASSICAL MECHANICS
SOLUTION SET #2

[1] Consider the standard map on the unit torus,

\[ x_{n+1} = x_n + y_n \mod 1 \]
\[ y_{n+1} = y_n + \kappa \sin(2\pi x_{n+1}) \mod 1. \]

Find all the fixed points and identify their stability as a function of the control parameter \( \kappa \).

[2] Write a computer program to iterate the map from problem [1]. For each value of \( \kappa \) you consider, iterate starting from \( N^2 \) initial conditions \((x_0, y_0) = (j/N, k/N)\), where \( j \) and \( k \) each run from 0 to \( N - 1 \). You can take \( N = 10 \).

(a) By experimenting, see if you can find the value of \( \kappa \) where there are no unbroken KAM tori which span the \( x \)-direction \( x \in [0, 1] \).

(b) Next, consider the standard map on the cylinder,

\[ x_{n+1} = x_n + y_n \mod 1 \]
\[ y_{n+1} = y_n + \kappa \sin(2\pi x_{n+1}) \mod 1, \]

where the \( y \) variable now may take values on the entire real line. For each given \( \kappa \), plot \( \langle y^2_n \rangle \) versus \( n \), where the average is over the \( N^2 \) initial conditions. Assuming the evolution is diffusive in the chaotic regime, compute the diffusion constant \( D(\kappa) \) from the formula \( \langle y^2_n \rangle = 2Dn \). Plot \( D(\kappa) \) versus \( \kappa \) over the range \( \kappa \in [1, 10] \). Compare to the value from the quasilinear approximation, \( D_{\text{ql}} = \frac{1}{4}\kappa^2 \).

[3] For the logistic map \( x_{n+1} = f(x_n) \) with \( f(x) = rx(1-x) \), plot the functions \( f^{(n)}(x) \) for \( n = 1, 2, \) and \( 3 \) and plot the intersections of \( y = f^{(n)}(x) \) with \( y = x \). Show how varying the control parameter \( r \) results in bifurcations corresponding to the appearance of 2-cycles and 4-cycles.