

Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - uv_x/c^2}$; $v'_y = \frac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \rightarrow -u$, primed \leftrightarrow unprimed; Doppler: $f' = f \sqrt{\frac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$
 $E = \sqrt{p^2 c^2 + m^2 c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 \text{ MeV}/c^2$; Proton: $m_p = 938.26 \text{ MeV}/c^2$; Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic unit: $1u = 931.5 \text{ MeV}/c^2$; electron volt: $1eV = 1.6 \times 10^{-19} \text{ J}$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc/\lambda - \phi$; $\phi =$ work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} \text{ m} \cdot \text{K}$

$I(T) = \int_0^\infty I(\lambda, T) d\lambda$; $I = (c/4)u$; $u(\lambda, T) = N(\lambda)E_{av}(\lambda, T)$; $N(\lambda) = \frac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E) dE$; $E_{av} = \frac{1}{N} \int_0^\infty EN(E) dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \frac{1}{N} \sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1,240 \text{ eV} \cdot \text{nm}$; $\lambda_m T = hc/4.96k$; $\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$

Boltzmann constant: $k = (1/11,604) \text{ eV/K}$; $1\text{\AA} = 1\text{A} = 0.1 \text{ nm}$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$; $\frac{h}{m_e c} = 0.0243 \text{ A}$

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

Uncertainty: $\Delta x \Delta k \sim 1$; $\Delta t \Delta \omega \sim 1$; $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

$\hbar c = 197.3 \text{ eV nm}$; group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$

Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \psi^* \psi = 1$

$P(x_1 < x < x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$; $\langle f(x) \rangle = \int_{-\infty}^{\infty} |\psi(x)|^2 f(x) dx$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$; $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 0.0381 eV nm^2$ (electron)

2D square well: $\Psi_{n_1 n_2}(x,y) = \Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $E_{n_1 n_2} = \frac{\pi^2 \hbar^2}{2m} \left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2}\right)$; $\Psi_{i,n}(w) = \sqrt{\frac{2}{L_i}} \sin\left(\frac{n\pi w}{L_i}\right)$

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n + \frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$

Step potential: reflection coef : $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha \Delta x}$; $T = e^{-2 \int_{x_1}^{x_2} \alpha(x) dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Rutherford scattering: $N(\theta) = \text{constant} \times \frac{1}{\sin^4(\theta/2)}$; $U = \frac{kzZe^2}{r}$; $ke^2 = e^2 / (4\pi\epsilon_0) = 1.44 eV \cdot nm$

Bohr atom: $r_n = (a_0 / Z)n^2$; $E_n = -E_0 Z^2 / n^2$; $E_0 = \frac{ke^2}{2a_0}$; $a_0 = \frac{\hbar^2}{m_e ke^2}$; $L = m_e v r = n\hbar$

$E_0 = 13.6 eV$; $a_0 = 0.0529 nm$

Problem 1 (6 points)

An electron in a harmonic oscillator potential has classical amplitude of oscillation $A=0.2\text{nm}$ when it is in its ground state.

- (a) What is its classical amplitude of oscillation when it is in the first excited state? Give your answer in nm.
(b) What is its ground state energy, in eV?

Problem 2 (6 points)

Electrons with kinetic energy 4eV have a 0.1% probability of tunneling through a rectangular potential barrier of length 0.3nm .

- (a) Find the height of the potential barrier, in eV.
(b) If the width of this barrier is reduced to 0.15nm , what is the probability that these electrons will tunnel through?

Problem 3 (6 points)

An alpha particle incident on a gold nucleus ($Z=79$) with impact parameter $b=0$ reaches a distance of closest approach to the center of the nucleus $2 \times 10^{-5}\text{nm}$. Assume that distance is larger than the nuclear radius.

- (a) What is the kinetic energy of this alpha particle (in MeV) when it is far from the nucleus?
(b) When this alpha particle is incident with an impact parameter b that is not zero, its distance of closest approach is $4 \times 10^{-5}\text{nm}$. What is its kinetic energy (in MeV) at that point?
(c) For extra credit: What was the value of the impact parameter b (in nm) for part (b)?
Hint: use angular momentum conservation.

Problem 4 (6 points)

Find (a) the shortest and (b) the longest wavelengths of the Lyman series (transitions to $n=1$) of doubly ionized lithium ($Z=3$). Give your answers in nm.

Problem 5 (6 points)

- (a) Find the energy of the photon emitted when an electron in a hydrogen atom makes a transition from the state $n=20$ to the state $n=19$, in eV. The frequency of that photon, in units eV/h , is that number.
(b) Find the classical frequencies of revolution for the electron in the state $n=20$ and in the state $n=19$, and the average of the two. Give all your answers in units eV/h . Verify that the average is close to the answer in (a). Hint: use angular momentum.