PHYSICS 2D QUIZ 3 FALL QUARTER 2022
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Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux/c^2)$

Velocity: $v_x' = \dfrac{v_x - u}{1 - uv_x/c^2}$; $v_y' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$; $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}}$

Inverse transformations: $u \to -u$, *primed* $\leftrightarrow$ *unprimed*; Doppler: $f' = f\sqrt{\dfrac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

$E = \sqrt{p^2c^2 + m^2c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 Mev/c^2$; Proton: $m_p = 938.26 Mev/c^2$; Neutron: $m_n = 939.55 Mev/c^2$

Atomic unit: $1u = 931.5 MeV/c^2$; electron volt: $1eV = 1.6 \times 10^{-19} J$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc/\lambda - \phi$; ϕ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} W/m^2 \cdot K^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} m \cdot K$

$I(T) = \int_0^\infty I(\lambda, T)\, d\lambda$; $I = (c/4)u$; $u(\lambda, T) = N(\lambda) E_{av}(\lambda, T)$; $N(\lambda) = \dfrac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = Ce^{-E/kT}$; $N = \int_0^\infty N(E)\, dE$; $E_{av} = \dfrac{1}{N}\int_0^\infty EN(E)\, dE$

Classical: $E_{av} = kT$; Planck: $E_n = n\varepsilon = nhf$; $N = \sum_{n=0}^\infty N(E_n)$; $E_{av} = \dfrac{1}{N}\sum_{n=0}^\infty E_n N(E_n)$

Planck: $E_{av} = \dfrac{hc/\lambda}{e^{hc/\lambda kT} - 1}$; $hc = 1{,}240 eV \cdot nm$; $\lambda_m T = hc/4.96k$; $\sigma = \dfrac{2\pi^5 k^4}{15c^2h^3}$

Boltzmann constant: $k = (1/11{,}604) eV/K$; 1Å=1A=0.1nm

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243A$

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Uncertainty: $\Delta x \Delta k \sim 1$ $\Delta t \Delta\omega \sim 1$; $\Delta x \Delta p \sim \hbar$ $\Delta t \Delta E \sim \hbar$; $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$

$\hbar c = 197.3$ eV nm ; group and phase velocity : $v_g = \dfrac{d\omega}{dk}$; $v_p = \dfrac{\omega}{k}$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(x,t)=\psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+U(x)\psi(x)=E\psi(x)$; $\int_{-\infty}^{\infty}dx\,\psi^*\psi=1$

$$P(x_1<x<x_2)=\int_{x_1}^{x_2}|\psi(x)|^2dx \quad ; \quad <f(x)>=\int_{-\infty}^{\infty}|\psi(x)|^2 f(x)dx$$

∞ square well: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n=\frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e}=0.0381eVnm^2$ (electron)

2D square well: $\Psi_{n_1n_2}(x,y)=\Psi_{1,n_1}(x)\Psi_{2,n_2}(y)$; $E_{n_1n_2}=\frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2})$; $\Psi_{i,n}(w)=\sqrt{\frac{2}{L_i}}\sin(\frac{n\pi w}{L_i})$

Problem 1 (6 points)
A free electron at time t=0 is described by the wavepacket

$$y(x)=A\frac{\cos(4nm^{-1}x)\sin(0.01nm^{-1}x)}{x}$$

(a) Make a qualitative graph of y(x) , indicate the uncertainty in the position of the electron, Δx, in this graph, and give the numerical value of Δx, in nm.
(b) Estimate the phase velocity of this electron, give your answer as its value divided by the speed of light. You can assume the electron is non-relativistic.

Problem 2 (6 points)
An electron is localized in a region of size 0.0001nm.
(a) From the uncertainty principle, find the uncertainty in its momentum, Δp, in units MeV/c.
(b) Using the result of (a) for Δp, estimate its kinetic energy in MeV.

Problem 3 (6 points)
An electron in a stationary state is described by the wavefunction:

$\psi(x)=A\sqrt{1-|x|}$ for $|x|\le 1$; $\psi(x)=0$ for $|x|>1$

(a) Find A. Justify your answer.
(b) Find approximately the probability that the electron is located in the region $-0.01\le x\le +0.01$. Justify your procedure.

Problem 4 (6 points)
An electron is in the lowest energy state (ground state) of an infinite potential energy well, and has energy 0.1e V.
(a) Find the length of the well L, in nm
(b) The electron absorbs an incoming photon and makes a transition to the next-lowest energy state (first excited state). Find the energy and wavelength of the photon, in eV and nm respectively.

<u>Problem 5</u> (6 points)
For the electron of Problem 4, find the probability that it is located in the region $0 \le x \le L/4$:
(a) After absorbing the photon.
(b) Before absorbing the photon.