Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: \( \Delta t = \gamma \Delta t_0 \); \( L = L_0 / \gamma \); \( c = 3 \times 10^8 \text{ m/s} \)

Lorentz transformation: \( x' = \gamma (x - ut) \); \( y' = y \); \( t' = \gamma (t - ux / c^2) \)

Velocity: \( v'_x = \frac{v_x - u}{1 - uv_x / c^2} \); \( v'_y = \frac{v_y}{\gamma (1 - uv_x / c^2)} \); \( \gamma = \frac{1}{\sqrt{1 - u^2 / c^2}} \)

Inverse transformations: \( u \rightarrow -u \), primed \( \leftrightarrow \) unprimed; Doppler: \( f' = \frac{f \sqrt{1 \pm u / c}}{1 \mp u / c} \)

Momentum: \( \vec{p} = \gamma m \vec{v} \); Energy: \( E = \gamma mc^2 \); Kinetic energy: \( K = (\gamma - 1)mc^2 \)

\( E = \sqrt{p^2 c^2 + m^2 c^4} \); rest energy: \( E_0 = mc^2 \)

Electron: \( m_e = 0.511\text{MeV} / c^2 \); Proton: \( m_p = 938.26\text{MeV} / c^2 \); Neutron: \( m_n = 939.55\text{MeV} / c^2 \)

Atomic unit: \( 1u = 931.5\text{MeV} / c^2 \); electron volt: \( 1eV = 1.6 \times 10^{-19} \text{J} \)

Photoelectric effect: \( eV_s = K_{\text{max}} = hf - \phi = hc / \lambda - \phi \); \( \phi = \) work function

Stefan law: \( I = \sigma T^4 \); \( \sigma = 5.67037 \times 10^{-8} \text{W} / \text{m}^2 \cdot \text{K}^4 \); Wien's law: \( \lambda_m T = 2.8978 \times 10^{-3} \text{m} \cdot K \)

\( I(T) = \int_0^\infty I(\lambda, T) d\lambda \); \( I = (c / 4)u \); \( u(\lambda, T) = N(\lambda)E_{av}(\lambda, T) \); \( N(\lambda) = \frac{8\pi}{\lambda^4} \)

Boltzmann distribution: \( N(E) = Ce^{-E/kT} \); \( N = \sum_{n=0}^\infty N(E_n) \); \( E_{av} = \frac{1}{N} \sum_{n=0}^\infty E_n N(E_n) \)

Classical: \( E_{av} = kT \); Planck: \( E_n = n\epsilon = nh\nu \); \( N = \sum_{n=0}^\infty N(E_n) \); \( E_{av} = \frac{1}{N} \sum_{n=0}^\infty E_n N(E_n) \)

Planck: \( E_{av} = \frac{hc / \lambda}{e^{hc / \lambda kT} - 1} \); \( hc = 1,240eV \cdot \text{nm} \); \( \lambda_m T = hc / 4.96k \); \( \sigma = \frac{2\pi^2 k^4}{15c^2 h^3} \)

Boltzmann constant: \( k = (1/11,604)eV / K \); \( 1\AA = 1\text{A} = 0.1\text{nm} \)

Compton scattering: \( \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos \theta) \); \( \frac{h}{m_e c} = 0.0243\text{A} \)

de Broglie: \( \lambda = \frac{h}{p} \); \( f = \frac{E}{h} \); \( \omega = 2\pi f \); \( k = \frac{2\pi}{\lambda} \); \( E = h\nu \); \( p = h\k \); \( E = \frac{p^2}{2m} \)

Uncertainty: \( \Delta x \Delta k \sim 1 \); \( \Delta t \Delta \omega \sim 1 \); \( \Delta x \Delta p \sim \hbar \); \( \Delta t \Delta E \sim \hbar \); \( \Delta A = \sqrt{< A^2 > - < A >^2} \)

\( hc = 197.3 \text{ eV nm} \); group and phase velocity: \( \nu_g = \frac{d\omega}{dk} \); \( \nu_p = \frac{\omega}{k} \)
Problem 1 (6 points)
A free electron at time $t=0$ is described by the wavepacket
t
$$y(x) = A \frac{\cos(4nm^{-1}x)}{x}$$

(a) Make a qualitative graph of $y(x)$, indicate the uncertainty in the position of the electron, $\Delta x$, in this graph, and give the numerical value of $\Delta x$, in nm.
(b) Estimate the phase velocity of this electron, give your answer as its value divided by the speed of light. You can assume the electron is non-relativistic.

Problem 2 (6 points)
An electron is localized in a region of size 0.0001nm.
(a) From the uncertainty principle, find the uncertainty in its momentum, $\Delta p$, in units MeV/c.
(b) Using the result of (a) for $\Delta p$, estimate its kinetic energy in MeV.

Problem 3 (6 points)
An electron in a stationary state is described by the wavefunction:
$$\psi(x) = A \frac{\cos(4nm^{-1}x)}{x} \text{ for } |x| \leq 1 \quad ; \quad \psi(x) = 0 \text{ for } |x| > 1$$

(a) Find $A$. Justify your answer.
(b) Find approximately the probability that the electron is located in the region $-0.01 \leq x \leq +0.01$. Justify your procedure.

Problem 4 (6 points)
An electron is in the lowest energy state (ground state) of an infinite potential energy well, and has energy $0.1eV$.
(a) Find the length of the well $L$, in nm
(b) The electron absorbs an incoming photon and makes a transition to the next-lowest energy state (first excited state). Find the energy and wavelength of the photon, in eV and nm respectively.
Problem 5 (6 points)
For the electron of Problem 4, find the probability that it is located in the region
\[ 0 \leq x \leq L/4 \]
(a) After absorbing the photon.
(b) Before absorbing the photon.