Problem 1

\[ \mu = 0.8C = \frac{4}{5} C \]

\[ \text{planet} \quad \overline{\text{planet}} \quad \text{Lo} = 200,000 \text{km} \]

The \( \gamma \) is: \( \gamma = \frac{1}{\sqrt{1 - \mu^2/c^2}} = \frac{1}{\sqrt{1 - 16/25}} = \frac{1}{\sqrt{\frac{9}{25}}} = \frac{5}{3} = \frac{5}{\sqrt{3}} \)

(a) For an observer on the spaceship, the length is contracted to:

\[ L = \frac{\text{Lo}}{\gamma} = 200,000 \text{km} \times \frac{3}{5} = \boxed{120,000 \text{ km}} \]

(b) For an observer on the spaceship, it is proper time \( \Delta t_0 \)

\[ \Delta t_0 = \frac{\Delta t}{\gamma} \]

\[ \Delta t = \frac{\text{Lo}}{\mu} = \frac{\text{Lo}}{0.8C} = \frac{5 \text{Lo}}{4C} \]

\[ \Delta t_0 = \frac{\Delta t}{\gamma} = \frac{5 \text{Lo}}{4C} \times \frac{3}{5} = \frac{3 \text{Lo}}{4C} = \frac{3}{4} \times \frac{200,000 \text{km}}{\text{300,000 km/s}} = 0.55 \]

Alternative reasoning: as seen from the spaceship,

\[ \Delta t_0 = \frac{L}{\mu} = \frac{\text{Lo}}{\gamma \mu} = \frac{\text{Lo}}{\frac{5}{3} \times \frac{4}{5} C} = \frac{3}{4} \frac{\text{Lo}}{C} = \text{same answer} \]
Problem 2

\[ L_0 = 1 \text{km} \quad \mu = 0.6c \]

\[ U = 0.8c \]

(b) Your speed relative to the train:

\[ U' = \frac{U - \mu}{\sqrt{1 - \mu^2/c^2}} = \frac{0.8c - 0.6c}{\sqrt{1 - 0.8 	imes 0.6}} = 0.385c \]

\[ \Rightarrow \frac{U'}{c} = 0.385 \]

(b) From an observer in the train, you are going by at speed \( U' \), so

\[ \Delta t' = \frac{L_0}{U'} = \frac{1 \text{km}}{0.385 \times 300,000 \text{ km/s}} = 8.667 \mu s \]
Problem 3

(a) Events A and B happen simultaneously in the train. That means, if light is emitted, it reaches C at the same time. An observer on the ground sees that the light reaches C at the same time.

Since the light travels at c in an observer on the ground, and the train moved to the right since the light was emitted, it must be emitted in A earlier, and in B later.

So the event in the back, A, happened first. An observer on the ground

(b) $t'_1 = 0 = t'_2$ is when the events occurred in frame $O'$

$x'_1 = 0$, $x'_2 = L_0$

In the ground frame:

$t = \gamma \left( t' + \frac{ux'}{c^2} \right)$

$t_1 = 0$,

$t_2 = \gamma \left( t'_2 + \frac{ux'_2}{c^2} \right) = \frac{\gamma L_0}{c^2} > 0$

$t_2 > 0 \Rightarrow$ the event in the front happened later, in agreement with (a).

$t_2 = \frac{5}{4} \cdot \frac{3}{8} \cdot \frac{L_0}{c} = \frac{5}{4} \times \frac{1 \text{ km}}{300,000 \text{ km/s}} = 2.5 \mu s$
Problem 4

\[ K = 0.511 \text{ MeV} = m_e c^2 = E_0 \]

\[ \Rightarrow \text{energy} \quad E = E_0 + K = 2m_e c^2 \]

From \[ E = \sqrt{p^2 c^2 + m_e^2 c^4} \]

\[ \Rightarrow p^2 c^2 = E^2 - m_e^2 c^4 \]

\[ \Rightarrow p^2 c^2 = 4m_e^2 c^4 - m_e^2 c^4 = 3m_e^2 c^4 \]

\[ \Rightarrow pc = \sqrt{3} m_e c^2 = 0.865 \text{ MeV} \quad (a) \]

(b) We can find \( u/c \) from:

\[ p = \gamma m u, \quad E = \gamma m c^2 \implies \frac{p}{E} = \frac{\gamma m u}{\gamma m c^2} = \frac{u}{c} \]

\[ \Rightarrow \frac{pc}{E} = \frac{u}{c} = \frac{\sqrt{3} m_e c^2}{2m_e c^2} = \frac{\sqrt{3}}{2} = 0.866 = \frac{u}{c} \quad (b) \]

Alternatively,

\[ E = \gamma m_e c^2 = 2m_e c^2 \implies \gamma = 2 \]

\[ \gamma = \frac{1}{\sqrt{1-u^2/c^2}} \quad \Rightarrow \frac{1}{\gamma^2} = 1 - \frac{u^2}{c^2} \quad \Rightarrow \frac{u^2}{c^2} = 1 - \frac{1}{\gamma^2} \]

\[ \Rightarrow \frac{u}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \]
Problem 5

Initial:
\[ p \rightarrow p \quad \leftrightarrow \quad 0 \rightarrow 0 \]

\[ p \quad p \quad p \quad p \quad \text{all at rest} \]

Initial energy:
\[ E_i = 2mpc^2 + 2K \]

Final energy:
\[ E_f = 4mpc^2 \]

\[ E_i = E_f \Rightarrow K = mpc^2 \]

\[ K = 938.26 \text{ MeV} \quad (a) \]

(b) We have the same situation as in problem 4, with \( K = E_0 \)

Therefore, the speed is the same as in problem 4:
\[ \frac{v}{c} = 0.866 \]

Note that momentum is conserved, \( p \rightarrow 0 \) initially and at the end.