

Justify all your answers to all problems. Write clearly (5 problems, one in the back)

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux / c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - uv_x / c^2}$; $v'_y = \frac{v_y}{\gamma(1 - uv_x / c^2)}$; $\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}$

Inverse transformations: $u \rightarrow -u$, primed \leftrightarrow unprimed; Doppler: $f' = f \sqrt{\frac{1 \pm u/c}{1 \mp u/c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$
 $E = \sqrt{p^2 c^2 + m^2 c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 \text{ MeV} / c^2$; Proton: $m_p = 938.26 \text{ MeV} / c^2$; Neutron: $m_n = 939.55 \text{ MeV} / c^2$

Atomic unit: $1u = 931.5 \text{ MeV} / c^2$; electron volt: $1eV = 1.6 \times 10^{-19} \text{ J}$

Problem 1 (6 points)

There are two planets a distance 200,000 km apart. A spaceship travels from one to the other at speed $0.8c$.

- (a) How distant are the planets as measured by an observer on the spaceship?
 (b) How long does the trip take according to an observer on the spaceship?

Give your answers in km and seconds for (a) and (b) respectively.

Problem 2 (6 points)

A 1 km long train (length measured in the train) is moving at speed $0.6c$ relative to the ground. You are on the ground, after the train goes by you, you start running in the same direction at speed $0.8c$.

- (a) What is your speed relative to the train? Give your answer as a factor times c .
 (b) As seen by an observer in the train, how long does it take you to go from the back to the front of the train? Give the answer in microseconds ($1\mu s = 10^{-6} \text{ s}$)

Problem 3 (6 points)

Inside a 1km long train moving at speed $0.6c$ with respect to the ground, two events happen simultaneously in the front and the back of the train, as seen by observers on the train.

- (a) Which event happened first according to observers on the ground? Explain your answer using the fact that the speed of light is the same in all reference frames.
Hint: imagine that the two events resulted in light being emitted, that would reach the center of the train simultaneously.
 (b) Using the Lorentz transformation, calculate the time interval between these two events according to observers on the ground. Give your answer in μs .

Problem 4 (6 points)

An electron is moving with kinetic energy 0.511 MeV .

- (a) Find its momentum p . Give the answer as pc in MeV .
 (b) Find its speed v . Give the answer as v/c .

Problem 5 (6 points)

As we discussed in class, the antiproton was discovered by having a proton accelerated with kinetic energy $\sim 6\text{GeV}=6,000\text{ MeV}$ impacting another proton at rest.

(a) If instead two protons are accelerated and undergo a head-on collision, what is the minimum kinetic energy that each proton must have so that an antiproton may be created? Give your answer in MeV.

The reaction is: $p + p \rightarrow p + p + \bar{p}$

(b) At what speed will the protons be moving in the laboratory frame before colliding? Give your answer as v/c .