1005 nm = (820.1 nm) \frac{n^2}{n^2 - 9} \\
1.225(n^2 - 9) = n^2

Solving, we find \( n = 7 \), so the transition connects the \( n = 7 \) and \( n = 3 \) states.

19. \( r_3 = 9a_0 = 9(0.0529 \text{ nm}) = 0.476 \text{ nm} \)

\[
v = \frac{nh}{mr} = c \frac{nhc}{mc^2r} = c \frac{3(1240 \text{ eV} \cdot \text{nm})/2\pi}{(0.511 \times 10^6 \text{ eV})(0.476 \text{ nm})} = 2.43 \times 10^{-3} c = 7.30 \times 10^5 \text{ m/s}
\]

\[
U = -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r} = -\frac{1.440 \text{ eV} \cdot \text{nm}}{0.476 \text{ nm}} = -3.02 \text{ eV}
\]

\[
K = \frac{e^2}{8\pi\varepsilon_0} \frac{1}{r} = \frac{1.440 \text{ eV} \cdot \text{nm}}{2(0.476 \text{ nm})} = 1.51 \text{ eV}
\]

20. The Lyman series consists of transitions from higher levels to the \( n = 1 \) level. The series limit would be the transition with the highest energy, corresponding to a jump from \( n_1 = \infty \) to \( n_2 = 1 \). The wavelength is found from Equation 6.33:

\[
\lambda = \frac{1}{R_n} \left( \frac{n_1^2 n_2^2}{n_1^2 - n_2^2} \right) = \frac{n_2^2}{R_n} \left( \frac{n_1^2}{n_1^2 - n_2^2} \right) = \frac{1}{1.09737 \times 10^7 \text{ m}^{-1}} = 91.13 \text{ nm}
\]

For the Paschen series \((n_2 = 3)\), the series limit is (with \( n_1 = \infty \))

\[
\lambda_{\text{limit}} = \frac{n_2^2}{R_n} = \frac{9}{1.09737 \times 10^7 \text{ m}^{-1}} = 820.1 \text{ nm}
\]

21. (a) From Equation 6.26, \( v = \frac{nh}{mr} = \frac{nh}{mn^2 a_0} \). Using Equation 6.29 for \( a_0 \), we obtain

\[
v = \frac{h}{nm(4\pi\varepsilon_0 \hbar^2/me^2)} = \frac{e^2}{4\pi\varepsilon_0 n\hbar} = \frac{\alpha c}{n}
\]

(b) When the nuclear charge is \( Ze \), we must replace \( e^2 \) with \( Ze^2 \), so \( v = Z\alpha c/n \).

22. The energy of the initial \( n = 5 \) state is \( E_5 = -13.6 \text{ eV} = -0.544 \text{ eV} \). An electron in this state can make transitions to any of the lower states with \( n = 4 \) \( (E_4 = -0.850 \text{ eV}) \), \( n = 3 \)
1005 nm = (820.1 nm) \frac{n^2}{n^2 - 9} \\
1.225(n^2 - 9) = n^2

Solving, we find \( n = 7 \), so the transition connects the \( n = 7 \) and \( n = 3 \) states.

19. \( r_3 = 9a_0 = 9(0.0529 \text{ nm}) = 0.476 \text{ nm} \)

\[
v = \frac{n\hbar}{mr} = c \frac{n\hbar/c}{m c^2 r} = c \frac{3(1240 \text{ eV} \cdot \text{nm})/2\pi}{(0.511 \times 10^6 \text{ eV})(0.476 \text{ nm})} = 2.43 \times 10^{-3} c = 7.30 \times 10^5 \text{ m/s}
\]

\[
U = -\frac{e^2}{4\pi \epsilon_0 r} = \frac{1.440 \text{ eV} \cdot \text{nm}}{0.476 \text{ nm}} = -3.02 \text{ eV}
\]

\[
K = \frac{e^2}{8\pi \epsilon_0 r} \frac{1.440 \text{ eV} \cdot \text{nm}}{2(0.476 \text{ nm})} = 1.51 \text{ eV}
\]

20. The Lyman series consists of transitions from higher levels to the \( n_2 = 1 \) level. The series limit would be the transition with the highest energy, corresponding to a jump from \( n_1 = \infty \) to \( n_2 = 1 \). The wavelength is found from Equation 6.33:

\[
\lambda = \frac{1}{R_{\infty}} \left( \frac{n_1^2 n_2^2}{n_1^2 - n_2^2} \right) = \frac{n_2^2}{R_{\infty}} \left( \frac{n_1}{n_1^2 - n_2^2} \right) = \frac{1}{1.09737 \times 10^7 \text{ m}^{-1}} = 91.13 \text{ nm}
\]

For the Paschen series \( (n_2 = 3) \), the series limit is (with \( n_1 = \infty \))

\[
\lambda_{\text{limit}} = \frac{n_2^2}{R_{\infty}} = \frac{9}{1.09737 \times 10^7 \text{ m}^{-1}} = 820.1 \text{ nm}
\]

21. (a) From Equation 6.26, \( v = \frac{n\hbar}{m n^2 a_0} \). Using Equation 6.29 for \( a_0 \), we obtain

\[
v = \frac{\hbar}{n m (4\pi \epsilon_0 \hbar^2 / m e^2)} = \frac{e^2}{4\pi \epsilon_0 n \hbar} \frac{1}{n} = \frac{\alpha c}{n}
\]

(b) When the nuclear charge is \( Z e \), we must replace \( e^2 \) with \( Z e^2 \), so \( v = Z\alpha c/n \).

22. The energy of the initial \( n = 5 \) state is \( E_s = -13.6 \text{ eV} = -0.544 \text{ eV} \). An electron in this state can make transitions to any of the lower states with \( n = 4 \) \( (E_4 = -0.850 \text{ eV}) \), \( n = 3 \)
\(E_3 = -1.51 \text{ eV}\), \(n = 2 \ (E_2 = -3.40 \text{ eV})\), and \(n = 1 \ (E_1 = -13.6 \text{ eV})\). The transition energies are:

\[
\begin{align*}
5 \rightarrow 4 &: \Delta E = E_5 - E_4 = -0.544 \text{ eV} - (-0.850 \text{ eV}) = 0.306 \text{ eV} \\
5 \rightarrow 3 &: \Delta E = E_5 - E_3 = -0.544 \text{ eV} - (-1.51 \text{ eV}) = 0.97 \text{ eV} \\
5 \rightarrow 2 &: \Delta E = E_5 - E_2 = -0.544 \text{ eV} - (-3.40 \text{ eV}) = 2.86 \text{ eV} \\
5 \rightarrow 1 &: \Delta E = E_5 - E_1 = -0.544 \text{ eV} - (-13.6 \text{ eV}) = 13.1 \text{ eV}
\end{align*}
\]

23. The Paschen series consists of transitions from higher levels that end in the \(n = 3\) level. The energies and wavelengths are:

\[
\begin{align*}
4 \rightarrow 3 &: \Delta E = (-13.60 \text{ eV}) \left(\frac{1}{4^2} - \frac{1}{3^2}\right) = 0.661 \text{ eV} \quad \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{0.661 \text{ eV}} = 1876 \text{ nm} \\
5 \rightarrow 3 &: \Delta E = (-13.60 \text{ eV}) \left(\frac{1}{5^2} - \frac{1}{3^2}\right) = 0.967 \text{ eV} \quad \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{0.967 \text{ eV}} = 1282 \text{ nm} \\
6 \rightarrow 3 &: \Delta E = (-13.60 \text{ eV}) \left(\frac{1}{6^2} - \frac{1}{3^2}\right) = 1.133 \text{ eV} \quad \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{1.133 \text{ eV}} = 1094 \text{ nm} \\
7 \rightarrow 3 &: \Delta E = (-13.60 \text{ eV}) \left(\frac{1}{7^2} - \frac{1}{3^2}\right) = 1.234 \text{ eV} \quad \lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{1.234 \text{ eV}} = 1005 \text{ nm}
\end{align*}
\]

The series limit is 1.511 eV, corresponding to a wavelength of 820.5 nm.

<table>
<thead>
<tr>
<th>Photon wavelength (nm)</th>
<th>1876</th>
<th>1282</th>
<th>1094</th>
<th>1005</th>
<th>820.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon energy (eV)</td>
<td>0.661</td>
<td>0.967</td>
<td>1.133</td>
<td>1.234</td>
<td>1.511</td>
</tr>
</tbody>
</table>

24. The photon energy of the incident light is

\[
E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{53.0 \text{ nm}} = 23.4 \text{ eV}
\]
When an atom in the ground state absorbs a 23.4-eV photon, the atom is ionized (which takes 13.6 eV). The excess energy, $23.4 \text{ eV} - 13.6 \text{ eV} = 9.8 \text{ eV}$, appears as the kinetic energy of the electron, which is now free of the atom. Neglecting a small recoil kinetic energy given to the proton, the electrons have a kinetic energy of 9.8 eV.

25. (a) The ionization energy is the magnitude of the energy of the electron. For the $n = 3$ level of hydrogen

$$|E_3| = \left| \frac{-13.6 \text{ eV}}{9} \right| = 1.51 \text{ eV}$$

(b) For singly ionized helium ($Z = 2$) we use Equation 6.38:

$$|E_n| = \left| \frac{(-13.6 \text{ eV})Z^2}{n^2} \right| = \left| \frac{(-13.6 \text{ eV})2^2}{2^2} \right| = 13.6 \text{ eV}$$

(c) $$|E_4| = \left| \frac{(-13.6 \text{ eV})Z^2}{n^2} \right| = \left| \frac{(-13.6 \text{ eV})3^2}{4^2} \right| = 7.65 \text{ eV}$$

26. (a) $E(4 \to 2) = E_4 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{2^2} \right) = 2.55 \text{ eV}$

$E(4 \to 3) = E_4 - E_3 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{3^2} \right) = 0.661 \text{ eV}$

$E(3 \to 2) = E_3 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{3^2} - \frac{1}{2^2} \right) = 1.89 \text{ eV}$

$E(4 \to 3) + E(3 \to 2) = 0.661 \text{ eV} + 1.89 \text{ eV} = 2.55 \text{ eV} = E(4 \to 2)$

(b) $E(4 \to 1) = E_4 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{1^2} \right) = 12.8 \text{ eV}$

$E(2 \to 1) = E_2 - E_1 = (-13.6 \text{ eV}) \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 10.2 \text{ eV}$

$E(4 \to 2) + E(2 \to 1) = 2.55 \text{ eV} + 10.2 \text{ eV} = 12.8 \text{ eV} = E(4 \to 1)$

27. The Lyman series consists of transitions that end in the $n = 1$ level. The smallest energy difference, corresponding to the longest wavelength, is $n = 2$ to $n = 1$. 
When an atom in the ground state absorbs a 23.4-eV photon, the atom is ionized (which takes 13.6 eV). The excess energy, 23.4 eV - 13.6 eV = 9.8 eV, appears as the kinetic energy of the electron, which is now free of the atom. Neglecting a small recoil kinetic energy given to the proton, the electrons have a kinetic energy of 9.8 eV.

25. (a) The ionization energy is the magnitude of the energy of the electron. For the \( n = 3 \) level of hydrogen

\[
|E_3| = \left| -\frac{13.6 \text{ eV}}{9} \right| = 1.51 \text{ eV}
\]

(b) For singly ionized helium \((Z = 2)\) we use Equation 6.38:

\[
|E_n| = \left| \frac{(-13.6 \text{ eV})Z^2}{n^2} \right| = \left| \frac{(-13.6 \text{ eV})2^2}{2^2} \right| = 13.6 \text{ eV}
\]

(c) \[
|E_4| = \left| \frac{(-13.6 \text{ eV})Z^2}{n^2} \right| = \left| \frac{(-13.6 \text{ eV})3^2}{4^2} \right| = 7.65 \text{ eV}
\]

26. (a) \( E(4 \rightarrow 2) = E_4 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{2^2} \right) = 2.55 \text{ eV} \)

\( E(4 \rightarrow 3) = E_4 - E_3 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{3^2} \right) = 0.661 \text{ eV} \)

\( E(3 \rightarrow 2) = E_3 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{3^2} - \frac{1}{2^2} \right) = 1.89 \text{ eV} \)

\( E(4 \rightarrow 3) + E(3 \rightarrow 2) = 0.661 \text{ eV} + 1.89 \text{ eV} = 2.55 \text{ eV} = E(4 \rightarrow 2) \)

(b) \( E(4 \rightarrow 1) = E_4 - E_2 = (-13.6 \text{ eV}) \left( \frac{1}{4^2} - \frac{1}{1^2} \right) = 12.8 \text{ eV} \)

\( E(2 \rightarrow 1) = E_2 - E_1 = (-13.6 \text{ eV}) \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 10.2 \text{ eV} \)

\( E(4 \rightarrow 2) + E(2 \rightarrow 1) = 2.55 \text{ eV} + 10.2 \text{ eV} = 12.8 \text{ eV} = E(4 \rightarrow 1) \)

27. The Lyman series consists of transitions that end in the \( n = 1 \) level. The smallest energy difference, corresponding to the longest wavelength, is \( n = 2 \) to \( n = 1 \).
\[ \Delta E = E_2 - E_1 = (-13.6 \text{ eV}) \cdot 2^2 \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 40.8 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{40.8 \text{ eV}} = 30.4 \text{ nm} \]

The largest energy difference would correspond to transitions from \( n = \infty \) to \( n = 1 \):

\[ \Delta E = E_\infty - E_1 = (-13.6 \text{ eV}) \cdot 2^2 \left( 0 - \frac{1}{1^2} \right) = 54.4 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{54.4 \text{ eV}} = 22.8 \text{ nm} \]

28. Using Equation 6.38, we have \( E_n = (-13.6 \text{ eV})Z^2/n^2 = (-54.4 \text{ eV})/n^2 \), so \( E_1 = -54.40 \text{ eV}, E_2 = -13.60 \text{ eV}, E_3 = -6.04 \text{ eV}, E_4 = -3.40 \text{ eV} \). The possible transitions are:

- 4 → 1: \( \Delta E = E_4 - E_1 = 51.00 \text{ eV} \) \( \lambda = hc / \Delta E = 24.31 \text{ nm} \)
- 4 → 2: \( \Delta E = E_4 - E_2 = 10.20 \text{ eV} \) \( \lambda = hc / \Delta E = 121.6 \text{ nm} \)
- 4 → 3: \( \Delta E = E_4 - E_3 = 2.64 \text{ eV} \) \( \lambda = hc / \Delta E = 469.7 \text{ nm} \)
- 3 → 1: \( \Delta E = E_3 - E_1 = 48.36 \text{ eV} \) \( \lambda = hc / \Delta E = 25.64 \text{ nm} \)
- 3 → 2: \( \Delta E = E_3 - E_2 = 7.56 \text{ eV} \) \( \lambda = hc / \Delta E = 164.0 \text{ nm} \)
- 2 → 1: \( \Delta E = E_2 - E_1 = 40.80 \text{ eV} \) \( \lambda = hc / \Delta E = 30.39 \text{ nm} \)
\[ \Delta E = E_2 - E_1 = (-13.6 \text{ eV})^2 \left( \frac{1}{2^2} - \frac{1}{1^2} \right) = 40.8 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{40.8 \text{ eV}} = 30.4 \text{ nm} \]

The largest energy difference would correspond to transitions from \( n = \infty \) to \( n = 1 \):

\[ \Delta E = E_\infty - E_1 = (-13.6 \text{ eV})^2 \left( 0 - \frac{1}{1^2} \right) = 54.4 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{54.4 \text{ eV}} = 22.8 \text{ nm} \]

28. Using Equation 6.38, we have \( E_n = (-13.6 \text{ eV})Z^2/n^2 = (-54.4 \text{ eV})/n^2 \), so \( E_1 = -54.40 \text{ eV}, E_2 = -13.60 \text{ eV}, E_3 = -6.04 \text{ eV}, E_4 = -3.40 \text{ eV} \). The possible transitions are:

- \( n = 4 \) to \( n = 1 \):  \( \Delta E = E_4 - E_1 = 51.00 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 24.31 \text{ nm} \)
- \( n = 4 \) to \( n = 2 \):  \( \Delta E = E_4 - E_2 = 10.20 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 121.6 \text{ nm} \)
- \( n = 4 \) to \( n = 3 \):  \( \Delta E = E_4 - E_3 = 2.64 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 469.7 \text{ nm} \)
- \( n = 3 \) to \( n = 1 \):  \( \Delta E = E_3 - E_1 = 48.36 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 25.64 \text{ nm} \)
- \( n = 3 \) to \( n = 2 \):  \( \Delta E = E_3 - E_2 = 7.56 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 164.0 \text{ nm} \)
- \( n = 2 \) to \( n = 1 \):  \( \Delta E = E_2 - E_1 = 40.80 \text{ eV} \quad \lambda = \frac{hc}{\Delta E} = 30.39 \text{ nm} \)
29. The gravitational force law is \( F = \frac{G m e m_p}{r^2} \) instead of \( F = \frac{e^2}{4 \pi \epsilon_0 r^2} \). The Bohr theory can thus be directly applied if we substitute \( G m e m_p \) for \( \frac{e^2}{4 \pi \epsilon_0} \). Equation 6.29 becomes

\[
a_o = \frac{\hbar^2}{G m^2 m_p} = \frac{(1.05 \times 10^{-34} \text{ J} \cdot \text{s})^2}{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(9.11 \times 10^{-31} \text{ kg})^2(1.67 \times 10^{-27} \text{ kg})} = 1.19 \times 10^{20} \text{ m}
\]

\[
E_2 - E_1 = \frac{m^2_e m^2_p}{2 \hbar^2} (G m e m_p)^2 \left[ \frac{1}{l^2} - \frac{1}{l^2} \right] = \frac{3G^2 m^2 e^2}{8 \hbar^2}
\]

\[
= \frac{3(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)^2(9.11 \times 10^{-31} \text{ kg})^3(1.67 \times 10^{-27} \text{ kg})^2}{8(1.05 \times 10^{-34} \text{ J} \cdot \text{s})^2} = 3.2 \times 10^{-97} \text{ J} = 2.0 \times 10^{-78} \text{ eV}
\]

30. (a) If the circumference is an integral number of de Broglie wavelengths \( 2 \pi r = n \lambda \), then after each orbit the waves will align, peak to peak and valley to valley, to give standing waves.

(b) \( 2 \pi r = n \lambda = n \frac{h}{p} = \frac{nh}{mv} \) so \( mv = \frac{nh}{2 \pi} = n \hbar \)

31. Let \( V_1 = 4 \text{ V}, V_2 = 7 \text{ V}, \) and \( V_3 = 9 \text{ V} \). Then decreases in the current should be observed at the following voltages:

\[
\begin{align*}
V_1 &= 4 \text{ V} & 3V_1 &= 12 \text{ V} & 2V_1 + V_3 &= 17 \text{ V} \\
V_2 &= 7 \text{ V} & V_1 + V_3 &= 13 \text{ V} & 2V_3 &= 18 \text{ V} \\
2V_1 &= 8 \text{ V} & 2V_2 &= 14 \text{ V} & 3V_1 + V_3 &= 19 \text{ V} \\
V_3 &= 9 \text{ V} & 2V_1 + V_2 &= 15 \text{ V} & 5V_1 &= 20 \text{ V} \\
V_1 + V_2 &= 11 \text{ V} & 4V_1 &= 16 \text{ V}
\end{align*}
\]

32. The energy difference between the ground state and the first excited state is

\[
E = \frac{\hbar c}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{590 \text{ nm}} = 2.10 \text{ eV}
\]

At \( V = 2.10 \text{ V} \), we expect to see a decrease in the current, as atoms are raised to the first excited state.

33. There are 4 peaks from 34 V to 80 V, so the excitation energy would be \((80-34)/4 \) or 11.5 eV. Energy levels for argon are listed by the National Institute of Standards and Technology (NIST) --

https://physics.nist.gov/PhysRefData/Handbook/Tables/argontable5.htm -- and show the first multiplet of excited states from wave numbers of 93,100 cm\(^{-1}\) (11.5 eV) to 95,400 cm\(^{-1}\) (11.8 eV), so clearly the Frank-Hertz apparatus must be exciting levels in this group. There are no levels below this group, so the de-excitation involves only a transition back to the ground state of about 11.6 eV (107 nm) in the ultraviolet region.
34. The energy uncertainty of a state with a lifetime of $10^{-8}$ s is

$$\Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58 \times 10^{-16} \text{ eV} \cdot \text{s}}{10^{-8} \text{ s}} = 6.58 \times 10^{-8} \text{ eV}$$

This energy uncertainty will be equal to the spacing $\Delta E = hf$ with $f$ given by Equation 6.43 when $n$ is large:

$$\Delta E = hf = \frac{me^4}{16\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^3} = \frac{mc^2}{(hc)} \left( \frac{e^2}{4\pi \epsilon_0} \right)^2 \frac{1}{n^3}$$

$$n = \sqrt[3]{\frac{mc^2(e^2/4\pi \epsilon_0)^2}{\Delta E(hc)^2}} = \sqrt[3]{\frac{(0.511 \times 10^6 \text{ eV})(1.440 \text{ eV} \cdot \text{nm})^2}{(6.58 \times 10^{-8} \text{ eV})(197 \text{ eV} \cdot \text{nm})^2}} = 746$$

$$r = n^2a_0 = (746)^2(5.29 \times 10^{-11} \text{ m}) = 29 \mu\text{m}$$

35. (a) The frequency of revolution is given by Equation 6.41:

$$f_n = \frac{me^4}{32\pi^3 \epsilon_0^2 \hbar^4} \frac{1}{n^3} = \frac{1}{\pi \hbar} \frac{me^4}{32\pi^2 \epsilon_0^2 \hbar^2} \frac{1}{n^3} = \frac{13.6 \text{ eV}}{\pi \hbar} \frac{1}{n^3} = \frac{6.58 \times 10^{15}}{n^3} \text{ Hz}$$

A similar calculation gives the radiation frequency from Equation 6.42:

$$f = \frac{me^4}{64\pi^4 \epsilon_0^2 \hbar^4} \frac{2n-1}{n^2(n-1)^2} = \frac{13.6 \text{ eV}}{2\pi \hbar} \frac{2n-1}{n^2(n-1)^2} = \frac{(6.58 \times 10^{15} \text{ Hz})}{2n-1} \frac{2n-1}{2n^2(n-1)^2}$$

For $n = 10$, we get $f_n = 6.58 \times 10^{12}$ Hz and $f = 7.72 \times 10^{12}$ Hz.

(b) For $n = 100$, $f_n = 6.58 \times 10^9$ Hz and $f = 6.68 \times 10^9$ Hz.

(c) For $n = 1000$, $f_n = 6.58 \times 10^6$ Hz and $f = 6.59 \times 10^6$ Hz.

(d) For $n = 10,000$, $f_n = 6.58 \times 10^3$ Hz and $f = 6.58 \times 10^3$ Hz. Note how $f$ approaches $f_n$ as $n$ becomes large, in accordance with the correspondence principle.

36. The Rydberg constant in ordinary hydrogen is

$$R_H = R_n \left( 1 + \frac{m}{M_H} \right) = R_n \left( 1 + \frac{5.48580 \times 10^{-4} \text{ u}}{1.007825 \text{ u}} \right) = R_n(1.000544)$$

and in “heavy” hydrogen or deuterium:
\[ R_\text{D} = R_\infty \left( 1 + \frac{m}{M_\text{D}} \right) = R_\infty \left( 1 + \frac{5.4858 \times 10^{-4} \text{ u}}{2.104102 \text{ u}} \right) = R_\infty (1.000272) \]

From Equation 6.33 the difference in wavelengths for the first line of the Balmer series \((n = 3 \text{ to } n = 2)\) is

\[ \lambda_\text{D} - \lambda_\text{H} = \left( \frac{1}{R_\text{D}} - \frac{1}{R_\text{H}} \right) \left( \frac{3^2 - 2^2}{3^2 - 2^2} \right) = \frac{7.2}{1.09737 \times 10^7 \text{ m}^{-1}} \left( \frac{1}{1.000272} - \frac{1}{1.000544} \right) = 0.178 \text{ nm} \]

This small wavelength difference led to the discovery of deuterium in 1931.

37. (a) 15 different transitions are possible: \(6 \to 5, 6 \to 4, 6 \to 3, 6 \to 2, 6 \to 1, 5 \to 4, 5 \to 3, 5 \to 2, 5 \to 1, 4 \to 3, 4 \to 2, 4 \to 1, 3 \to 2, 3 \to 1, 2 \to 1\).
(b) Only 5 of the transitions change \(n\) by one unit.
(c) One.

38. (a) From the \(n = 8\) level, downward transitions are possible to any level of smaller \(n\). The transitions with the longest wavelengths are those with the smallest energy differences.

\[ \Delta E = E_8 - E_7 = (-13.6 \text{ eV})Z^2 \left( \frac{1}{8^2} - \frac{1}{7^2} \right) = 0.260 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.260 \text{ eV}} = 4.77 \mu\text{m} \]

\[ \Delta E = E_8 - E_6 = (-13.6 \text{ eV})Z^2 \left( \frac{1}{8^2} - \frac{1}{6^2} \right) = 0.661 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.661 \text{ eV}} = 1.88 \mu\text{m} \]

\[ \Delta E = E_8 - E_5 = (-13.6 \text{ eV})Z^2 \left( \frac{1}{8^2} - \frac{1}{5^2} \right) = 1.33 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{1.33 \text{ eV}} = 0.935 \mu\text{m} \]

(b) The transition with the shortest wavelength is the one with the largest energy difference.

\[ \Delta E = E_8 - E_1 = (-13.6 \text{ eV})Z^2 \left( \frac{1}{8^2} - \frac{1}{1^2} \right) = 53.6 \text{ eV} \]

\[ \lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{53.6 \text{ eV}} = 23.2 \text{ nm} \]

(c) From the \(n = 8\) level, the atom can absorb a photon and the electron will jump to a state of larger \(n\). The longest absorption wavelengths correspond to the smallest energy differences.