\[ A_2 = A \frac{\cos \frac{\pi}{3}}{\cos \phi} = 0.661A \]

3. \[ E_1 = \frac{h^2}{8mL^2} = 5.6 \text{ eV} \]

With \( L' = 2L \),
\[ E'_1 = \frac{h^2}{8mL'^2} = \frac{h^2}{8m(2L)^2} = \frac{1}{4} \frac{h^2}{8mL^2} = \frac{1}{4} (5.6 \text{ eV}) = 1.4 \text{ eV} \]

4. With \( \lambda_n = 2L/n \),
\[ \lambda_1 = \frac{2(0.144 \text{ nm})}{1} = 0.288 \text{ nm} \]
\[ \lambda_2 = \frac{2(0.144 \text{ nm})}{2} = 0.144 \text{ nm} \]
\[ \lambda_3 = \frac{2(0.144 \text{ nm})}{3} = 0.096 \text{ nm} \]

5. The smallest energy is (using Equation 5.3)
\[ E_1 = \frac{h^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.062 \text{ nm})^2} = 98 \text{ eV} \]

Then \( E_2 = 2^2 E_1 = 391 \text{ eV} \) and \( E_3 = 3^2 E_1 = 881 \text{ eV} \).

6. With \( L = 1.2 \times 10^{-14} \text{ m} = 10 \text{ fm} \),
\[ E_1 = \frac{h^2}{8mL^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{8(940 \text{ MeV})(12 \text{ fm})^2} = 1.4 \text{ MeV} \]

7. (a) At \( x = a \), \( \psi_1 = \psi_2 \) and \( d\psi_1/dx = d\psi_2/dx \):
\[ 0 = (a - d)^2 - c \quad \text{and} \quad -2ab = 2(a - d) \]
3. \[ E_1 = \frac{h^2}{8mL^2} = 5.6 \text{ eV} \]

With \( L' = 2L \),

\[ E'_1 = \frac{h^2}{8mL'^2} = \frac{h^2}{8m(2L)^2} = \frac{1}{4} \frac{h^2}{8mL^2} = \frac{1}{4} (5.6 \text{ eV}) = 1.4 \text{ eV} \]

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\[ \lambda_1 = \frac{2(0.144 \text{ nm})}{1} = 0.288 \text{ nm} \quad \lambda_2 = \frac{2(0.144 \text{ nm})}{2} = 0.144 \text{ nm} \quad \lambda_3 = \frac{2(0.144 \text{ nm})}{3} = 0.096 \text{ nm} \]

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7. (a) At \( x = a \), \( \psi_1 = \psi_2 \) and \( d\psi_1 / dx = d\psi_2 / dx \):

\[ 0 = (a - d)^2 - c \quad \text{and} \quad -2ab = 2(a - d) \]
From the second equation, \( d = a(b + 1) \). Inserting this into the first equation, we find 
\( c = a^2b^2 \).

(b) With \( \psi_2 = \psi_3 \) at \( x = w \), we get \((w - d)^2 - c = 0\), or

\[
w = d + \sqrt{c} = a(b + 1) + \sqrt{a^2b^2} = a(2b + 1)
\]
The slope is discontinuous at \( w \) suggesting an infinite discontinuity in the potential energy at that location.

8. (a) The regions with \( x < a \) and \( x > a \) do not contribute to the normalization. The normalization integral is

\[
\int |\psi(x)|^2 \, dx = \int_{-a}^{a} b^2 (a^2 - x^2)^2 \, dx = b^2 \int_{-a}^{a} (a^4 - 2a^2x^2 + x^4) \, dx = b^2 \left( a^4x - 2a^2x^3 + \frac{x^5}{5} \right)_{-a/2}^{a/2}
\]
Evaluating the integral and setting it equal to 1, we find

\[
b^2 \left( 2a^5 - \frac{4a^5}{3} + \frac{2a^5}{5} \right) = 1 \quad \text{or} \quad b = \sqrt[5]{\frac{15}{16a^5}}
\]

(b) \( P(x) \, dx = |\psi(x)|^2 \, dx = b^2 (a^2 - x^2)^2 \, dx \), and with \( x = +a/2 \) and \( dx = 0.010a \) we obtain

\[
P(x) \, dx = \frac{15}{16a^5} \left( \frac{a^2}{4} - a^2 \right)^2 (0.010a) = 0.0053
\]

(c) \( P(a/2 : a) = \int_{a/2}^{a} |\psi(x)|^2 \, dx = \int_{a/2}^{a} b^2 (a^2 - x^2)^2 \, dx = \frac{15}{16a^5} \left( a^4x - 2a^2x^3 + \frac{x^5}{5} \right)_{a/2}^{a}
\]
\[
= \frac{15}{16a^5} \left[ a^4 \left( a - \frac{a}{2} \right) - \frac{2a^2}{3} \left( a^3 - \frac{a^3}{8} \right) + \frac{1}{5} \left( a^5 - \frac{a^5}{32} \right) \right] = 0.104
\]

9. With \( \psi(x) = Cxe^{-bx} \), we have \( d\psi / dx = Ce^{-bx} - bCxe^{-bx} \) and

\[
\frac{d^2\psi}{dx^2} = -2bCe^{-bx} + b^2Cxe^{-bx}
\]
We now substitute \( \psi(x) \) and \( d^2\psi / dx^2 \) into the Schrödinger equation:
\[ -\frac{\hbar^2}{2m}(-2bCe^{-bx} + b^2Cxe^{-bx}) + U(x)Cxe^{-bx} = ECxe^{-bx} \]

Canceling the common factor of \( Ce^{-bx} \) and solving for \( E \),

\[ E = \frac{\hbar^2 b}{mx} - \frac{\hbar^2 b^2}{2m} + U(x) \]

The energy \( E \) will be a constant only if the two terms that depend on \( x \) cancel each other:

\[ \frac{\hbar^2 b}{mx} + U(x) = 0 \quad \text{or} \quad U(x) = -\frac{\hbar^2 b}{mx} \]

The cancellation of the two terms depending on \( x \) leaves only the remaining term for the energy:

\[ E = -\frac{\hbar^2 b^2}{2m} \]

10. (a) The regions with \( x < -L/2 \) and \( x > +L/2 \) do not contribute to the normalization integral. The remaining integral is:

\[
\int_{-L/2}^{0} |\psi(x)|^2 \, dx = \int_{-L/2}^{0} C^2 (2x / L + 1)^2 \, dx + \int_{0}^{L/2} C^2 (-2x / L + 1)^2 \, dx
\]

\[ = C^2 \left( 4x^3 / 3L^2 + 2x^2 / L + x \right)_{-L/2}^{0} + C^2 \left( 4x^3 / 3L^2 - 2x^2 / L + x \right)_{0}^{L/2} = C^2 L / 3 \]

Setting the integral equal to 1 gives \( C = \sqrt{3/L} \).

(b) \( P(x) \, dx = |\psi(x)|^2 \, dx = C^2 (4x^2 / L^2 - 4x/L + 1) \, dx \) and with \( x = 0.250L \) and \( dx = 0.010L \),

\[ P(x) \, dx = \frac{3}{L} \left( \frac{4}{L^2} \frac{L^2}{16} - \frac{4}{L} \frac{0.010L}{4} + 1 \right) (0.010L) = 0.0075 \]

(c) \( P(0 : L/4) = \int_{0}^{L/4} |\psi(x)|^2 \, dx = C^2 \int_{0}^{L/4} \left( \frac{4}{L^2} x^2 - \frac{4}{L} x + 1 \right) \, dx = \frac{3}{L} \left( \frac{4}{L^2} \frac{x^3}{3} - \frac{4}{L} \frac{x^2}{2} + x \right)_{0}^{L/4} = \frac{1}{2} \)

(d) \( \langle x \rangle = \int |\psi(x)|^2 \, x \, dx = \frac{3}{L} \int_{-L/2}^{0} \left( \frac{4x^2}{L^2} + \frac{4x}{L} + 1 \right) \, x \, dx + \int_{0}^{L/2} \left( \frac{4x^2}{L^2} - \frac{4x}{L} + 1 \right) \, x \, dx \)
\[
\frac{3}{L} \left( \frac{x^4}{L^2} + \frac{4x^3}{3L} + \frac{x^2}{2} \right)_0^{L/2} + \frac{3}{L} \left( \frac{x^4}{L^2} - \frac{4x^3}{3L} + \frac{x^2}{2} \right)_0^{L/2} = 0
\]

It is apparent from the shape of the wave function that the equal probability densities for positive and negative \( x \) cancel to give an average of zero.

\[
\langle x^2 \rangle = \int |\psi(x)|^2 x^2 \, dx = \frac{3}{L} \left( \frac{4x^2}{L^2} + \frac{4x}{L} + 1 \right)_0^{L/2} x^2 \, dx + \int_0^{L/2} \left( \frac{4x^2}{L^2} - \frac{4x}{L} + 1 \right) x^2 \, dx
\]

\[
= \frac{3}{L} \left( \frac{4x^5}{5L^2} + \frac{x^4}{L} + \frac{x^3}{3} \right)_0^{L/2} + \frac{3}{L} \left( \frac{4x^5}{5L^2} - \frac{x^4}{L} + \frac{x^3}{3} \right)_0^{L/2} = \frac{L}{40}
\]

The rms value is then \( x_{\text{rms}} = \sqrt{\langle x^2 \rangle} = \sqrt{L^2 / 40} = 0.158L \).

11. (a) The normalization integral is

\[
\int_{-\infty}^{+\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{+\infty} A^2 x^2 e^{-2bx} \, dx = 2A^2 \int_0^{+\infty} x^2 e^{-2bx} \, dx = 2A^2 \frac{2}{(2b)} = 1
\]

so \( A = \sqrt{2b} \).

(b) The wave function can be written as \( \psi(x) = Ae^{bx} \) for \( x < 0 \) and \( \psi(x) = Ae^{-bx} \) for \( x > 0 \).

\[
\int_{-\infty}^{+\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} A^2 e^{2bx} \, dx + \int_{0}^{+\infty} A^2 e^{-2bx} \, dx = \frac{2A^2}{2b}
\]

and \( A = \sqrt{b} \).

12. For the \( x > 0 \) wave function, we have

\[
\frac{dy}{dx} = - Abe^{-bx} \quad \text{and} \quad \frac{d^2y}{dx^2} = Ab^2e^{-bx}
\]

Substituting into the Schrödinger equation then gives

\[
-\frac{\hbar^2}{2m} Ab^2e^{-bx} + UAe^{-bx} = EAe^{-bx} \quad \text{or} \quad -\frac{\hbar^2b^2}{2m} + U = E
\]

This is consistent with a constant value of \( U \), which we can take to be 0, giving \( E = -\hbar^2b^2 / 2m \). Repeating the calculation for the \( x < 0 \) wave function gives an identical result.
So the potential energy is a constant (zero) for \( x < 0 \) and for \( x > 0 \). What happens at \( x = 0 \)? Note that the wave function is continuous at \( x = 0 \) (both give \( \psi = A \) at \( x = 0 \)) but that the derivative \( d\psi/dx \) is not continuous. This suggests an infinite discontinuity in \( U(x) \) at \( x = 0 \), and because the wave functions approach 0 as \( x \to \infty \) there must be a negative potential energy that produces the bound states. So the potential energy is

\[
U(x) = 0 \quad x < 0, \ x > 0
\]

\[
U(x) = -\infty \quad x = 0
\]

This type of function is known as a delta function.

13. (a) \( E_1 = \frac{\hbar^2}{8ml^2} = \frac{(hc)^2}{8(mc^2)L^2} = \frac{(1240 \text{ eV nm})^2}{8(511,000 \text{ eV})(0.285 \text{ nm})^2} = 4.63 \text{ eV} \)

\( 4 \to 1 \): \( \Delta E = E_4 - E_1 = 16E_1 - E_1 = 15E_1 = 15(4.63 \text{ eV}) = 69.5 \text{ eV} \)

(b) \( 4 \to 3 \): \( \Delta E = E_4 - E_3 = 16E_1 - 9E_1 = 7E_1 = 7(4.63 \text{ eV}) = 32.4 \text{ eV} \)
\( 4 \to 2 \): \( \Delta E = E_4 - E_2 = 16E_1 - 4E_1 = 12E_1 = 12(4.63 \text{ eV}) = 55.6 \text{ eV} \)
\( 3 \to 2 \): \( \Delta E = E_3 - E_2 = 9E_1 - 4E_1 = 5E_1 = 5(4.63 \text{ eV}) = 23.2 \text{ eV} \)
\( 3 \to 1 \): \( \Delta E = E_3 - E_1 = 9E_1 - E_1 = 8E_1 = 8(4.63 \text{ eV}) = 37.0 \text{ eV} \)
\( 2 \to 1 \): \( \Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 = 3(4.63 \text{ eV}) = 13.9 \text{ eV} \)

14. With \( E_1 = 1.54 \text{ eV} \) and \( E_n = n^2E_1 \) we have

\( \Delta E_3 = E_3 - E_1 = 9E_1 - E_1 = 8E_1 = 8(1.54 \text{ eV}) = 12.3 \text{ eV} \)

\( \Delta E_4 = E_4 - E_1 = 16E_1 - E_1 = 15E_1 = 15(1.54 \text{ eV}) = 23.1 \text{ eV} \)

15. \( \int_0^L A^2 \sin^2 \frac{n\pi x}{L} \, dx = A^2 \frac{L}{n\pi} \int_0^{n\pi} \sin^2 u \, du \) with \( u = n\pi x / L \). The integral is a standard form that can be found in integral tables:

\[
A^2 \frac{L}{n\pi} \left( \frac{n\pi}{2} \sin^2 u \right)_0^{n\pi} = A^2 \frac{L}{2}
\]

Setting the integral equal to 1 for normalization gives \( A^2 L/2 = 1 \) or \( A = \sqrt{2/L} \).
16. (a)  
\[ P(0 : L/3) = \int_0^{L/3} |\psi_1(x)|^2 \, dx = \int_0^{L/3} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_0^{\pi/3} \sin^2 u \, du \]
\[ = \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \bigg|_0^{\pi/3} = 0.1955 \]

(b)  
\[ P(L/3 : 2L/3) = \int_{L/3}^{2L/3} |\psi_1(x)|^2 \, dx = \int_{L/3}^{2L/3} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_{\pi/3}^{2\pi/3} \sin^2 u \, du \]
\[ = \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \bigg|_{\pi/3}^{2\pi/3} = 0.6090 \]

(c)  
\[ P(2L/3 : L) = \int_{2L/3}^{L} |\psi_1(x)|^2 \, dx = \int_{2L/3}^{L} \frac{2}{L} \sin^2 \frac{\pi x}{L} \, dx = \frac{2}{\pi} \int_{2\pi/3}^{\pi} \sin^2 u \, du \]
\[ = \frac{2}{\pi} \left( \frac{u}{4} - \frac{\sin 2u}{4} \right) \bigg|_{2\pi/3}^{\pi} = 0.1955 \]

17. (a)  
\[ P(x)dx = |\psi_2(x)|^2 \, dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \sin^2 \frac{3\pi (0.188 \text{ nm})}{0.189 \text{ nm}} 0.001 \text{ nm} = 2.63 \times 10^{-5} \]

(b)  
\[ P(x)dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \sin^2 \frac{3\pi (0.031 \text{ nm})}{0.189 \text{ nm}} 0.001 \text{ nm} = 0.0106 \]

(c)  
\[ P(x)dx = \frac{2}{L} \sin^2 \frac{3\pi x}{L} \, dx = \frac{2}{0.189 \text{ nm}} \sin^2 \frac{3\pi (0.079 \text{ nm})}{0.189 \text{ nm}} 0.001 \text{ nm} = 5.42 \times 10^{-3} \]

(d) A classical particle has a uniform probability to be found anywhere within the region, so \( P(x)dx = (0.001 \text{ nm})/(0.189 \text{ nm}) = 5.29 \times 10^{-3} \).

18. With \( E = E_0(n_x^2 + n_y^2) \) the levels above \( 50E_0 \) are as follows:

<table>
<thead>
<tr>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4</td>
<td>( 52E_0 )</td>
<td>6</td>
<td>5</td>
<td>( 61E_0 )</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>( 52E_0 )</td>
<td>5</td>
<td>6</td>
<td>( 61E_0 )</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>( 53E_0 )</td>
<td>7</td>
<td>4</td>
<td>( 65E_0 )</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>( 53E_0 )</td>
<td>4</td>
<td>7</td>
<td>( 65E_0 )</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>( 58E_0 )</td>
<td>8</td>
<td>1</td>
<td>( 65E_0 )</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>( 58E_0 )</td>
<td>1</td>
<td>8</td>
<td>( 65E_0 )</td>
</tr>
</tbody>
</table>
The level at \( E = 65E_0 \) is 4-fold degenerate.

19. With \( E = E_0 (n_x^2 + n_y^2) / 4 \) the levels are as follows:

<table>
<thead>
<tr>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
<th>( n_x )</th>
<th>( n_y )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>( 1.25E_0 )</td>
<td>2</td>
<td>3</td>
<td>( 6.25E_0 )</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>( 2.00E_0 )</td>
<td>1</td>
<td>5</td>
<td>( 7.25E_0 )</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( 2.25E_0 )</td>
<td>2</td>
<td>4</td>
<td>( 8.00E_0 )</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>( 3.25E_0 )</td>
<td>3</td>
<td>1</td>
<td>( 9.25E_0 )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>( 5.00E_0 )</td>
<td>1</td>
<td>6</td>
<td>( 10.00E_0 )</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>( 5.00E_0 )</td>
<td>3</td>
<td>2</td>
<td>( 10.00E_0 )</td>
</tr>
</tbody>
</table>

The levels at \( E = 5.00E_0 \) and \( E = 10.00E_0 \) are both 2-fold degenerate.

20. Using Equations 5.39 and 5.40, we have

\[
\frac{\partial \psi}{\partial x} = g(y) \frac{df}{dx} = g(y)(k_x A \cos k_x x - k_x B \sin k_x x)
\]

\[
\frac{\partial^2 \psi}{\partial x^2} = g(y) \frac{d^2 f}{dx^2} = g(y)(-k_x^2 A \sin k_x x - k_x^2 B \cos k_x x) = -k_x^2 g(y) f(x)
\]

\[
\frac{\partial \psi}{\partial y} = f(x) \frac{dg}{dy} = f(x)(k_y C \cos k_y y - k_y D \sin k_y y)
\]

\[
\frac{\partial^2 \psi}{\partial y^2} = f(x) \frac{d^2 g}{dy^2} = f(x)(-k_y^2 C \sin k_y y - k_y^2 D \cos k_y y) = -k_y^2 f(x) g(y)
\]

With \( U(x, y) = 0 \) inside the well, Equation 5.37 gives

\[
-\frac{\hbar^2}{2m} \left[ -k_x^2 f(x) g(y) - k_y^2 f(x) g(y) \right] = Ef(x) g(y)
\]

and so \( E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2) \).
35. (a) \[ E = n^2 E_1 = 10^2 \frac{h^2}{8mL^2} = \frac{100(hc)^2}{8mc^2L^2} = \frac{100(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.132 \text{ nm})^2} = 2160 \text{ eV} \]

(b) \[ \Delta p = \sqrt{p^2} = \sqrt{2mE} = \frac{1}{c} \sqrt{2mc^2E} = \frac{1}{c} \sqrt{2(511,000 \text{ eV})(2160 \text{ eV})} = 4.70 \times 10^4 \text{ eV}/c \]

(c) \[ \Delta x \sim \frac{\hbar}{\Delta p} = \frac{\hbar}{2\pi c \Delta p} = \frac{1}{2\pi} \frac{1240 \text{ eV} \cdot \text{nm}}{4.70 \times 10^4 \text{ eV}} = 4.2 \times 10^{-3} \text{ nm} \]

36.

37. \[ (x^2)_{av} = \int_0^L |\psi(x)|^2 x^2 dx = \frac{2}{L} \int_0^L x^2 \sin^2 \frac{n\pi x}{L} dx = \frac{2}{L} \left( \frac{L}{n\pi} \right)^3 \int_0^{n\pi} u^2 \sin u du \quad \text{with} \quad u = \frac{n\pi x}{L} \]

The integral is a standard form that can be found in integral tables.

\[ (x^2)_{av} = \frac{2L^2}{(n\pi)^3} \left[ \frac{u^3}{6} - \left( \frac{u^2}{4} - \frac{1}{8} \right) \sin 2u - \frac{u \cos 2u}{4} \right]_0^{n\pi} = \frac{2L^2}{(n\pi)^3} \left[ \frac{(n\pi)^3}{6} - \frac{n\pi}{4} \right] = L^2 \left( \frac{1}{3} - \frac{1}{2n^2\pi^2} \right) \]

38. With \( x_{av} = L/2 \) from Example 5.5, we have
35. (a) \[ E = n^2 E_i = 10^2 \frac{\hbar^2}{8mL^2} = \frac{100(\hbar c)^2}{8mc^2 L^2} = \frac{100(1240 \text{ eV} \cdot \text{nm})^2}{8(511,000 \text{ eV})(0.132 \text{ nm})^2} = 2160 \text{ eV} \]

(b) \[ \Delta p = \sqrt{p^2} = \sqrt{2mE} = \frac{1}{c} \sqrt{2mc^2 E} = \frac{1}{c} \sqrt{2(511,000 \text{ eV})(2160 \text{ eV})} = 4.70 \times 10^4 \text{ eV/c} \]

(c) \[ \Delta x \sim \frac{\hbar}{2\pi c\Delta p} = \frac{1}{2\pi} \frac{\hbar c}{4.70 \times 10^4 \text{ eV}} = 4.2 \times 10^{-3} \text{ nm} \]

36.

37. \[ (x^2)_{av} = \frac{1}{L} \int_0^L |\psi(x)|^2 x^2 dx = \frac{2}{L} \int_0^L x^2 \sin^2 \frac{n\pi x}{L} dx = \frac{2}{L} \left( \frac{L}{n\pi} \right)^3 \int_0^{n\pi} u^2 \sin u du \quad \text{with} \quad u = \frac{n\pi x}{L} \]

The integral is a standard form that can be found in integral tables.

\[ (x^2)_{av} = \frac{2L^2}{(n\pi)^3} \left[ \frac{u^3}{6} - \left( \frac{u^2}{4} - \frac{1}{8} \right) \sin 2u - \frac{u \cos 2u}{4} \right]_0^{n\pi} = \frac{2L^2}{(n\pi)^3} \left[ \frac{(n\pi)^3}{6} - \frac{n\pi}{4} \right] = L^2 \left( \frac{1}{3} - \frac{1}{2n^2 \pi^2} \right) \]

38. With \( x_{av} = L/2 \) from Example 5.5, we have
\[ \Delta x = \sqrt{(x^2)_{av} - (x_{av})^2} = \sqrt{\left( \frac{1}{3} \frac{1}{2n^2 \pi^2} \right) - \left( \frac{L}{2} \right)^2} = \frac{L}{\sqrt{12}} \frac{1}{2n^2 \pi^2} \]

39. (a) The particle has no preferred direction of motion, so it is equally likely to be moving in the positive and negative \( x \) directions. We therefore expect that \( p_{av} = 0 \).
(b) Because the potential energy is zero inside the well, the kinetic energy is equal to the total energy:

\[ K = E_n \quad \text{or} \quad \frac{p^2}{2m} = \frac{h^2 n^2}{8mL^2} \quad \text{so} \quad p^2 = \frac{h^2 n^2}{4L^2} \]

For a given level \( n \), \( p^2 \) is constant so \( (p^2)_{av} \) has that same value.

(c) \( \Delta p = \sqrt{(p^2)_{av} - (p_{av})^2} = \sqrt{\frac{h^2 n^2}{4L^2} - 0} = \frac{hn}{2L} \)

40. \( \frac{d\psi}{dx} = A \frac{d}{dx} xe^{-ax^2} = A(e^{-ax^2} - 2ax^2 e^{-ax^2}) = Ae^{-ax^2}(1 - 2ax^2) \)

\( \frac{d^2\psi}{dx^2} = A((-4ax)e^{-ax^2} - (1 - 2ax^2)(-2ax)e^{-ax^2}) = Ae^{-ax^2}(-6ax + 4a^2 x^3) \)

Substituting the second derivative into the Schrödinger equation, we have

\[ -\frac{h^2}{2m} Ae^{-ax^2}(-6ax + 4a^2 x^3) + 2kx^2 Axe^{-ax^2} = EAxe^{-ax^2} \]

After canceling common factors and combining terms,

\[ x^2 \left( \frac{k}{2} - \frac{2h^2 a^2}{m} \right) + \left( \frac{3ah^2}{m} - E \right) = 0 \]

In order for this to be valid for all possible values of \( x \), both of the quantities in parentheses must be zero:

\[ \frac{k}{2} = \frac{2h^2 a^2}{m} \quad \text{or} \quad a = \frac{\omega_0 m}{2h} \quad \text{AND} \quad E = \frac{3ah^2}{m} = \frac{3h^2 \omega_0 m}{2h^2} = \frac{3}{2} h \omega_0 \]

\[ \int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = A^2 \int_{-\infty}^{\infty} x^2 e^{-2ax^2} \, dx = 2A^2 \int_{0}^{\infty} x^2 e^{-2ax^2} \, dx = 2A^2 \frac{A^2}{\sqrt{8\pi}} = \frac{A^4}{\sqrt{2 \pi}} \frac{1}{4} \]

where we have made the substitution \( u = x\sqrt{2a} \) to put the integral into a standard form that is found in integral tables. Setting the result equal to 1 gives