Chapter 1

1. (a) Conservation of momentum gives $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{H}}v_{\text{H,initial}} + m_{\text{He}}v_{\text{He,initial}} = m_{\text{H}}v_{\text{H,final}} + m_{\text{He}}v_{\text{He,final}}$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$v_{\text{He,final}} = \frac{m_{\text{H}}(v_{\text{H,initial}} - v_{\text{H,final}})}{m_{\text{He}}}$$

$$= \frac{(1.674 \times 10^{-27} \text{ kg})[1.1250 \times 10^7 \text{ m/s} - (-6.724 \times 10^6 \text{ m/s})]}{6.646 \times 10^{-27} \text{ kg}} = 4.527 \times 10^6 \text{ m/s}$$

(b) Kinetic energy is the only form of energy we need to consider in this elastic collision. Conservation of energy then gives $K_{\text{initial}} = K_{\text{final}}$, or

$$\frac{1}{2}m_{\text{H}}v_{\text{H,initial}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He,initial}}^2 = \frac{1}{2}m_{\text{H}}v_{\text{H,final}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He,final}}^2$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$v_{\text{He,final}} = \sqrt{\frac{m_{\text{H}}(v_{\text{H,initial}}^2 - v_{\text{H,final}}^2)}{m_{\text{He}}}}$$

$$= \sqrt{\frac{(1.674 \times 10^{-27} \text{ kg})[(1.1250 \times 10^7 \text{ m/s})^2 - (-6.724 \times 10^6 \text{ m/s})^2]}{6.646 \times 10^{-27} \text{ kg}}} = 4.527 \times 10^6 \text{ m/s}$$

2. (a) Let the helium initially move in the $x$ direction. Then conservation of momentum gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}; \quad m_{\text{He}}v_{\text{He,initial}} = m_{\text{He}}v_{\text{He,final}} \cos \theta_{\text{He}} + m_{\text{O}}v_{\text{O,final}} \cos \theta_{\text{O}}$$

$$p_{y,\text{initial}} = p_{y,\text{final}}; \quad 0 = m_{\text{He}}v_{\text{He,final}} \sin \theta_{\text{He}} + m_{\text{O}}v_{\text{O,final}} \sin \theta_{\text{O}}$$

From the second equation,

$$v_{\text{O,final}} = -\frac{m_{\text{He}}v_{\text{He,final}} \sin \theta_{\text{He}}}{m_{\text{O}} \sin \theta_{\text{O}}} = -\frac{(6.6465 \times 10^{-27} \text{ kg})(6.636 \times 10^6 \text{ m/s})(\sin 84.7^\circ)}{(2.6560 \times 10^{-26} \text{ kg})[\sin(-40.4^\circ)]} = 2.551 \times 10^6 \text{ m/s}$$

(b) From the first momentum equation,
5. (a) The kinetic energy of the electrons is

\[ K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(9.11\times10^{-31} \text{ kg})(1.76\times10^6 \text{ m/s})^2 = 14.11\times10^{-19} \text{ J} \]

In passing through a potential difference of \( \Delta V = V_f - V_i = +4.15 \text{ volts} \), the potential energy of the electrons changes by

\[ \Delta U = q\Delta V = (-1.602\times10^{-19} \text{ C})(+4.15 \text{ V}) = -6.65\times10^{-19} \text{ J} \]

Conservation of energy gives \( K_i + U_i = K_f + U_f \), so

\[ K_f = K_i + (U_i - U_f) = K_i - \Delta U = 14.11\times10^{-19} \text{ J} + 6.65\times10^{-19} \text{ J} = 20.76\times10^{-19} \text{ J} \]

\[ v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(20.76\times10^{-19} \text{ J})}{9.11\times10^{-31} \text{ kg}}} = 2.13\times10^6 \text{ m/s} \]

(b) In this case \( \Delta V = -4.15 \text{ volts} \), so \( \Delta U = +6.65 \times 10^{-19} \text{ J} \) and thus

\[ K_f = K_i - \Delta U = 14.11\times10^{-19} \text{ J} - 6.65\times10^{-19} \text{ J} = 7.46\times10^{-19} \text{ J} \]

\[ v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(7.46\times10^{-19} \text{ J})}{9.11\times10^{-31} \text{ kg}}} = 1.28\times10^6 \text{ m/s} \]

6. (a) \( \Delta x_A = v\Delta t_A = (0.624)(2.997\times10^8 \text{ m/s})(124\times10^{-9} \text{ s}) = 23.2 \text{ m} \)

(b) \( \Delta x_B = v\Delta t_B = (0.624)(2.997\times10^8 \text{ m/s})(159\times10^{-9} \text{ s}) = 29.7 \text{ m} \)

7. With \( T = 35^\circ \text{C} = 308 \text{ K} \) and \( P = 1.22 \text{ atm} = 1.23\times10^5 \text{ Pa} \),

\[ \frac{N}{V} = \frac{P}{kT} = \frac{1.23\times10^5 \text{ Pa}}{(1.38\times10^{-23} \text{ J/K})(308 \text{ K})} = 2.89\times10^{25} \text{ atoms/m}^3 \]

so the volume available to each atom is \( (2.89 \times 10^{25}/\text{m}^3)^{-1} = 3.46 \times 10^{-26} \text{ m}^3 \). For a spherical atom, the volume would be

\[ \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(0.710\times10^{-10} \text{ m})^3 = 1.50\times10^{-30} \text{ m}^3 \]

The fraction is then

\[ \frac{1.50\times10^{-30}}{3.46\times10^{-26}} = 4.34\times10^{-5} \]
\[ p_{x,\text{initial}} = p_{x,\text{final}} : m_1v_1 = m_2v'_2 \cos \theta \quad \text{or} \quad v = 3v'_2 \cos \theta \]
\[ p_{y,\text{initial}} = p_{y,\text{final}} : 0 = m_1v'_1 - m_2v'_2 \sin \theta \quad \text{or} \quad v'_1 = 3v'_2 \sin \theta \]

We first solve for the speeds by eliminating \( \theta \) from these equations. Squaring the two momentum equations and adding them, we obtain \( v^2 + v'_1^2 = 9v'_2^2 \), and combining this result with the energy equation allows us to solve for the speeds:

\[ v'_1 = v / \sqrt{2} \quad \text{and} \quad v'_2 = v / \sqrt{6} \]

By substituting this value of \( v'_2 \) into the first momentum equation, we obtain

\[ \cos \theta = \sqrt{2/3} \quad \text{or} \quad \theta = 35.3^\circ \]

12. The combined particle, with mass \( m' = m_1 + m_2 = 3m \), moves with speed \( v' \) at an angle \( \theta \) with respect to the \( x \) axis. Conservation of momentum then gives:

\[ p_{x,\text{initial}} = p_{x,\text{final}} : m_1v_1 = m'v' \cos \theta \quad \text{or} \quad v = 3v' \cos \theta \]
\[ p_{y,\text{initial}} = p_{y,\text{final}} : m_2v_2 = m'v' \sin \theta \quad \text{or} \quad \frac{4}{3}v = 3v' \sin \theta \]

We can first solve for \( \theta \) by dividing these two equations to eliminate the unknown \( v' \):

\[ \tan \theta = \frac{4}{3} \quad \text{or} \quad \theta = 53.1^\circ \]

Now we can substitute this result into either of the momentum equations to find

\[ v' = 5v / 9 \]

The kinetic energy lost is the difference between the initial and final kinetic energies:

\[ K_{\text{initial}} - K_{\text{final}} = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 - \frac{1}{2}m'v'^2 = \frac{1}{2}mv^2 + \frac{1}{2}(2m)(\frac{4}{3}v)^2 - \frac{1}{2}(3m)(\frac{4}{3}v)^2 = \frac{20}{3}(\frac{1}{2}mv^2) \]

The total initial kinetic energy is \( \frac{1}{2}mv^2 + \frac{1}{2}(2m)(\frac{4}{3}v)^2 = \frac{12}{9}(\frac{1}{2}mv^2) \). The loss in kinetic energy is then \( \frac{20}{31} = 51\% \) of the initial kinetic energy.

13. (a) Let \( v_1 \) represent the helium atom that moves in the \(+x\) direction, and let \( v_2 \) represent the other helium atom (which might move either in the positive or negative \( x \) direction). Then conservation of momentum \( (p_{x,\text{initial}} = p_{x,\text{final}}) \) gives

\[ mv = m_1v_1 + m_2v_2 \quad \text{or} \quad 2v = v_1 + v_2 \]
\[
\cot 30^\circ = \frac{v'_1 \cos \phi + v}{v'_1 \sin \phi}
\]

which can be solved to give \( \phi = 53.8^\circ \). Using this value of \( \phi \), we can then find \( v_1 = 2.41 \times 10^6 \) m/s. We can also write the velocity addition equations for \( m_2 \):

\[
v_2 \cos \theta = -v'_1 \cos \phi + v \quad \text{and} \quad v_2 \sin \theta = -v'_1 \sin \phi
\]

which describe respectively the \( x \) and \( y \) components. Solving as we did for \( m_1 \), we find \( v_2 = 1.25 \times 10^6 \) m/s and \( \theta = 74.9^\circ \).

15. (a) With \( K = \frac{1}{2} kT \),

\[
\Delta K = \frac{1}{2} k \Delta T = \frac{1}{2} (1.38 \times 10^{-23} \text{ J/K})(80 \text{ K}) = 1.66 \times 10^{-21} \text{ J} = 0.0104 \text{ eV}
\]

(b) With \( U = mgh \),

\[
h = \frac{U}{mg} = \frac{1.66 \times 10^{-21} \text{ J}}{(40.0 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})(9.80 \text{ m/s}^2)} = 2550 \text{ m}
\]

16. We take \( dE \) to be the width of this small interval: \( dE = 0.04kT - 0.02kT = 0.02kT \), and we evaluate the distribution function at an energy equal to the midpoint of the interval \( (E = 0.03kT) \):

\[
\frac{dN}{N} = \frac{N(E)dE}{N} = \frac{2}{\sqrt{\pi} (kT)^{3/2}} \left(0.03kT\right)^{1/2} e^{-\left(0.03kT)/kT\right} (0.02kT) = 3.79 \times 10^{-3}
\]

17. If we represent the molecule as two atoms considered as point masses \( m \) separated by a distance \( 2R \), the rotational inertia about one of the axes is \( I_x = mR^2 + mR^2 = 2mR^2 \). On average, the rotational kinetic energy about any one axis is \( \frac{1}{2} kT \), so

\[
\frac{1}{2} I_x \omega_x^2 = \frac{1}{2} kT \quad \text{and}
\]

\[
\omega_x = \sqrt{\frac{kT}{I_x}} = \sqrt{\frac{kT}{2mR^2}} = \sqrt{\frac{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})}{2(15.9995 \text{ u})(1.6605 \times 10^{-27} \text{ kg/u})(0.0605 \times 10^{-9} \text{ m})^2}} = 4.61 \times 10^{12} \text{ rad/s}
\]