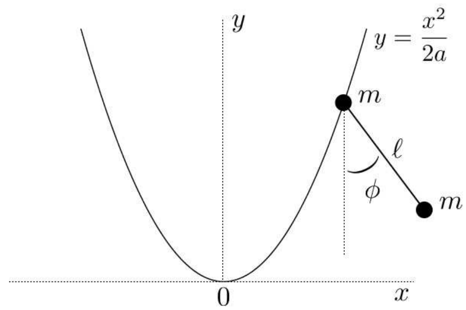
PHYSICS 110A : MECHANICS 1
PROBLEM SET #10

[**1**] Recall problem #3 from HW #6, in which a mass m moves frictionlessly under the influence of gravity along the curve $y = x^2/2a$. Attached to the mass is a massless rigid rod of length ℓ, at the end of which is an identical mass m. The rod is constrained to swing in the (x, y) plane, as depicted in the figure below.



(a) Choose as generalized coordinates x and ϕ. Find the kinetic energy T and potential energy U.

(b) Choose as generalized displacements the coordinates $\eta_1 = x$ and $\eta_2 = \phi$ themselves. For small oscillations, find the T and V matrices. It may be convenient to define $\Omega_1 \equiv \sqrt{g/a}$ and $\Omega_2 \equiv \sqrt{g/\ell}$.

(c) Find the eigenfrequencies of the normal modes of oscillation.

(d) Suppose $\Omega_1 = \sqrt{3}\,\Omega_0$ and $\Omega_2 = 2\,\Omega_0$, where Ω_0 has dimensions of frequency. Find the modal matrix.

[**2**] Two pendula each consisting of a point mass m hanging from a massless rigid rod of length ℓ are coupled by a massless spring of spring constant k (between the mass points). When the pendula hang vertically, the spring is unstretched. Compute the eigenfrequencies and the normal modes. Classify the normal modes according to whether they are even or odd with respect to the group $\mathbb{Z}_2$, generated by the elements $\mathbb{I}$ (identity) and P (reflection about a vertical line midway between the two pendula).

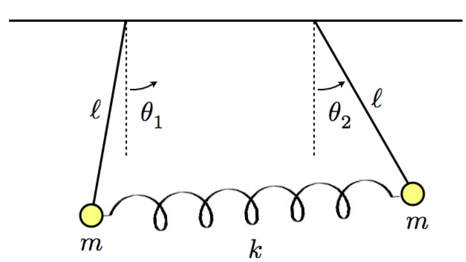


Figure 1: Coupled identical pendula.

[3] Two masses m_1 and m_2 are connected to a spring and a pendulum arm, as depicted in fig. 2. The unstretched length of the spring is a.

(a) Choosing generalized coordinates x and θ as shown, write the Lagrangian for this system.

(b) Expanding about equilibrium, write the Lagrangian for small oscillations as a quadratic form. *It may be convenient to define the generalized displacements* $\eta_1 \equiv x$ *and* $\eta_2 \equiv \ell\theta$.

(c) Let $\varOmega \equiv \sqrt{k/m_1}$, $\nu \equiv \sqrt{g/\ell}$, and $r \equiv m_2/m_1$. Find the T and V matrices.

(d) Find the eigenfrequencies $\omega_{1,2}$.

(e) Find an expression for the ratios of the components of the normal mode eigenvectors $\psi_2^{(+)}/\psi_1^{(+)}$ and $\psi_2^{(-)}/\psi_1^{(-)}$.

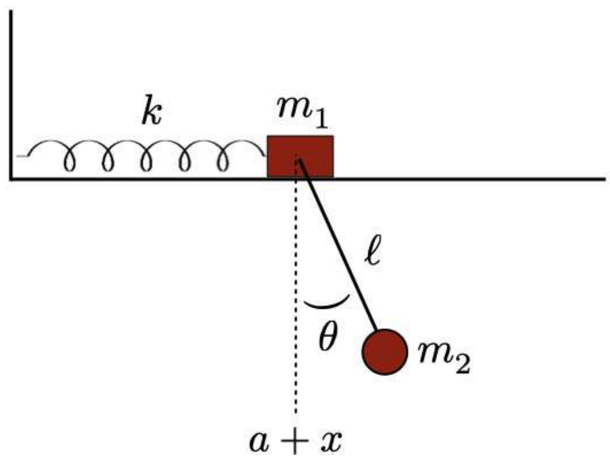


Figure 2: A spring, a pendulum, and two masses.