PHYSICS 110A : MECHANICS 1
PROBLEM SET #7

[**1**] A point mass m slides inside a hoop of radius R and mass M, which itself rolls without slipping on a horizontal surface, as depicted in fig. 1.

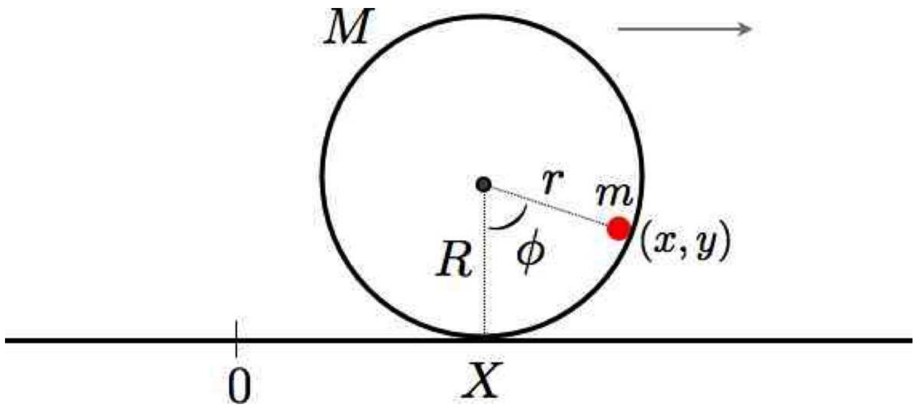

Figure 1: A mass point m rolls inside a hoop of mass M and radius R which rolls without slipping on a horizontal surface.

Choose as general coordinates (X, ϕ, r), where X is the horizontal location of the center of the hoop, ϕ is the angle the mass m makes with respect to the vertical ($\phi = 0$ at the bottom of the hoop), and r is the distance of the mass m from the center of the hoop. Since the mass m slides inside the hoop, there is a constraint:

$$G(X, \phi, r) = r - R = 0 \ .$$

Nota bene: The kinetic energy of the moving hoop, including translational and rotational components (but not including the mass m), is $T_{\text{hoop}} = M\dot{X}^2$ (*i.e.* twice the translational contribution alone).

(a) Find the Lagrangian $L(X, \phi, r, \dot{X}, \dot{\phi}, \dot{r}, t)$.

(b) Find all the generalized momenta p_σ, the generalized forces F_σ, and the forces of constraint Q_σ.

(c) Derive expressions for all conserved quantities.

(d) Derive a differential equation of motion involving the coordinate $\phi(t)$ alone. *I.e.* your equation should not involve r, X, or the Lagrange multiplier λ.

[**2**] *Warning: challenging!* A uniformly dense ladder of mass m and length 2ℓ leans against a block of mass M, as shown in Fig. 2. Choose as generalized coordinates the horizontal

position X of the right end of the block, the angle θ the ladder makes with respect to the floor, and the coordinates (x, y) of the ladder's center-of-mass. These four generalized coordinates are not all independent, but instead are related by a certain set of constraints.

Recall that the kinetic energy of the ladder can be written as a sum $T_{\text{CM}} + T_{\text{rot}}$, where $T_{\text{CM}} = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$ is the kinetic energy of the center-of-mass motion, and $T_{\text{rot}} = \frac{1}{2}I\dot{\theta}^2$, where I is the moment of inertial. For a uniformly dense ladder of length 2ℓ, $I = \frac{1}{3}m\ell^2$.

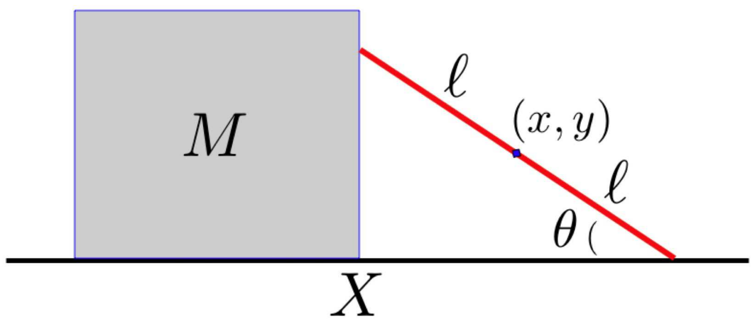

Figure 2: A ladder of length 2ℓ leaning against a massive block. All surfaces are frictionless..

(a) Write down the Lagrangian for this system in terms of the coordinates X, θ, x, y, and their time derivatives.

(b) Write down all the equations of constraint.

(c) Write down all the equations of motion.

(d) Find all conserved quantities.

(e) What is the condition that the ladder detaches from the block? You do not have to solve for the angle of detachment! Express the detachment condition in terms of any quantities you find convenient.