PHYSICS 110A : MECHANICS 1
PROBLEM SET #4

[1] An electrical circuit consists of a resistor $R$ and a capacitor $C$ connected in series to an emf $V(t)$.

(a) Write down the differential equation for the charge $Q(t)$ on one of the capacitor plates.

(b) Solve the homogeneous equation for $Q(t)$, i.e. find $Q(t)$ when $V(t) = 0$ subject to arbitrary initial value of $Q(0)$.

(c) Solve for the current $I(t)$ flowing in the circuit when $V(t) = V_0 \Theta(t)$. Assume $Q(0) = 0$.

(d) Solve for $I(t)$ when $V(t) = V_0 \sin(\Omega t) \Theta(t)$ and $Q(0)=0$.

For parts (c) and (d), you should use the Green’s function formalism in the time domain. The following integral may prove useful:

\[
\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{e^{-i\omega s}}{1 - i\omega \tau} = \frac{1}{\tau} e^{-s/\tau} \Theta(s)
\]

[2] A forced, damped harmonic oscillator obeys the equation of motion

\[
\left(\frac{d}{dt} + \alpha\right)\left(\frac{d}{dt} + \beta\right)x = f_0 e^{-\gamma t} \Theta(t)
\]

Compute $x(t)$ assuming $x(0) = \dot{x}(0) = 0$. 