1. Introduction to Dynamics (Sept. 23 and 26)
   essential elements of dynamics
   discrete and differential equations
   deterministic versus stochastic
   dynamical systems
   examples

2. Motion in $d = 1$: Two-Dimensional Phase Flows (Sept. 28)
   $(x, v)$ phase space
   dynamical system $\frac{d}{dt} \{ x \ v \} = \{ a(x,v) \}$
   two-dimensional phase flows
   examples: harmonic oscillator and pendulum
   fixed points in two-dimensional phase space; separatrices

3. Solution of the Equations of One-Dimensional Motion (Sept. 30 and Oct. 3)
   potential energy $U(x)$
   conservation of energy
   sketching phase flows from $U(x)$
   solution by quadratures
   turning points; period of orbit

4. Linear Oscillations (Oct. 5)
   Taylor’s theory and the ubiquity of harmonic motion
   the damped harmonic oscillator: $\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = 0$
     reduction to algebraic equation
     generalization to all autonomous homogeneous linear ODEs
   solution to the damped harmonic oscillator: underdamped and overdamped behavior

5. Forced Linear Oscillations (Oct. 7)
   $\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = f(t)$
   solution for harmonic forcing $f(t) = A \cos(\Omega t)$
   presence of homogeneous solution: transients
   amplitude resonance and phase lag; Q factor

6. Green’s Functions for Autonomous Linear ODEs (Oct. 10)
   Fourier transform
   physical meaning of $G(t - t')$; causality
   response to a pulse
7. Systems of Particles (Oct. 12 and 14)
  kinetic, potential, and interaction potential energies
  forces; Newton’s third law
  momentum conservation
  torque and angular momentum
  kinetic energy and the work-energy theorem

8. MIDTERM EXAMINATION (Oct. 17)

9. Calculus of Variations I (Oct. 19)
  Snell’s law for refraction at an interface
  continuum limit of many interfaces
  functionals
  variational calculus: extremizing \( \int dx \, L(y, y', x) \)
  preview: Newton’s second law from \( L = T - U \)

10. Calculus of Variations II (Oct. 21 and 24)
  Examples
    surfaces of revolution
    geodesics
    brachistochrone
  generalization to several dependent and independent variables
  Constrained Extremization
  Lagrange undetermined multipliers in calculus: review
  systems with integral constraints
    hanging rope of fixed length
  holonomic constraints

11. Lagrangian Dynamics (Oct. 26 and 28)
  generalized coordinates
  action functional
  equations of motion: Newton’s second law
  examples: spring, pendulum, etc.
  double pendulum: Lagrangian and equations of motion
  Lagrangian for a charged particle interacting with an electromagnetic field
  Lorentz force law

12. Noether’s Theorem and Conservation Laws (Oct. 31 and Nov. 2)
  continuous symmetries
  “one-parameter family of diffeomorphisms” \( q_i \to h_i^\lambda(q_1, \ldots, q_N) \)
  Noether’s theorem and the conserved “charge” \( Q = \sum_i \frac{\partial L}{\partial \dot{q}_i} \frac{\partial h_i^\lambda}{\partial \lambda} \bigg|_{\lambda=0} \)
  linear and angular momentum

13. Constrained Dynamical Systems (Nov. 4 and 7)
  undetermined multipliers as forces of constraints
  simple pendulum with \( r = l \) or \( x^2 + y^2 = l^2 \) constraint
  Examples
15. The Two-Body Central Force Problem (Nov. 9, 11, and 14)
CM and relative coordinates
angular momentum conservation and Kepler’s law $\dot{A} = \text{const.}$
ergy conservation
the effective potential
radial equation of motion for the relative coordinate
the effective potential and its interpretation
phase curves
solution for $r(t)$ and $\phi(t)$ by quadratures

16. The Shape of the Orbit (Nov. 16 and 18)
equation for $r(\phi)$, the geometric shape of the orbit
$s = 1/r$ substitution
examples
almost circular orbits: bound versus closed motion, precession

17. Coupled Oscillations I: The Double Pendulum (Nov. 21 and 23)
review: Lagrangian for the double pendulum
equations of motion
linearization
solution of two coupled linear equations
normal modes

18. Coupled Oscillations II: General Theory (Nov. 25 and 28)
harmonic potentials
$T$ and $V$ matrices
normal modes
the mathematical problem: simultaneous diagonalization of $T$ and $V$

19. Coupled Oscillations III: The Recipe (Nov. 30 and Dec. 2)
eigenvalues: $\det(\omega^2 T - V) = 0$
eigenvectors: $(\omega^2 T_{ij} - V_{ij})a_j^{(s)} = 0$
ormalization: $a_i^{(s)} T_{ij} a_j^{(s')} = \delta_{ss'}$
modal matrix: $A_{js} = a_j^{(s)}$
examples

• COMPREHENSIVE FINAL EXAMINATION (Dec. 6)