Physics 218a

Lecture 7: Momentum Transport and Intrinsic Rotation; with ITBS

i) Loose Ends

ii) Additional Readings (available Thursday, PDT)

- Annotated version of previous lecture notes; gaps filled in.

- Annotated notes on Predator-Prey Model (HUST - Phil '20 - '21)

- "Feedback for Physicists" - J. Bechhofer
  [important!]

and see also:

- R. May, "Stability and Complexity of Model Ecosystems I" (highly recommended) -> simple, useful book

+ posted articles.
ii) Questions

- Relation to 'converse cascade'?
- Constraint of strong magnetic field?

In 2D fluid ($D<4V> = 0$) drag - control

\[ \Delta_0 \langle \Delta_\phi \rangle + \rho \phi \times A \Delta_\phi = - \mu \nabla^2 \phi + \gamma \nabla^2 \phi + \frac{1}{2} \nabla \times \nabla \times \phi \]

2 conservation laws

- (quadratic) - energy
- (cubic) - vorticity

\[ \left\{ \begin{array}{l}
\text{Energy} = \int d^3 x \langle \Delta \phi \rangle^2 / 2 \\
\text{Enstrophy} = \int d^3 x \langle \nabla \times \phi \rangle^2 / 2
\end{array} \right. \]

So if we start at some intermediate scale $\ell_0$, have 2 ranges of self-similar transfer.

\[ \text{Inverse energy cascade range} \quad (\text{Kraichnan '67}) \]

\[ \text{Forward Enstrophy cascade range} \]

(See Fluids notes)
here: “cascade” = self-similar transfer

c.d. For averse cascade range

$$E_s = \frac{(\nu \lambda)}{7}$$

Rate of energy transfer

$$E(k) = E_s k^{-7/3}$$

- Spectrum grows from y-ls
- continue to box

Note:

- Inverse cascade is continuous transfer
  (c.e. flow in scale space)

- should be contrasted to disparate scale interaction
Reynolds stress modulation

\[ \frac{\zeta F}{\zeta D} \]

Shearing

Non-local transfer in \( k \)-space

"Inverse cascade" claimed as a mystery, but bi-coherence experiments indicate

\[ \zeta F - \zeta D \text{ transfer is non-local} \]

Primary of strong-B constraint is fundamental to both mechanisms, but

beyond - quite different.

\( \rightarrow \) Flow Damping - \( Z_{\text{org}} \) (Tokamak)

- Depends on collisionality (can)

- Banana / Plateau - Emotion of flow with (stationary)

\[ \sim E \text{ Vicinity trapped particles.} \]
- Pfirsch-Schluter (very edge, only)

Magnetic pumping - collisions + geometry

and

\[ n^2, \partial n^2 \rightarrow \text{scale?} \]

and

Charge exchange - neutral friction (scale independent)

Other:

- IF goes unstable \( \rightarrow \) "tertiary instability"

how quantify vorticity transport?

\[ \langle \nabla \cdot \mathbf{v} \rangle \]

\( \rightarrow \) expect a turbulent viscosity

tertiary "seen" in some computer simulations (not in experiment, i.e. tokamak).
- On flow develops to allow wave - flow resonance

\[ \frac{\partial}{\partial t} \langle U^2 \phi \rangle = \frac{\partial}{\partial r} \langle U r \phi^2 \rangle + \text{nonlinear damping} \]

then, on flow:

\[ \frac{\partial^2 \phi}{\partial r^2} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\omega}{c} \phi \right) + \frac{1}{c} \left( \omega - \omega_0 \right) \]

\[ \text{resonance with ExB flow} \]

\[ \langle U r \phi^2 \rangle = - \sum \frac{V_{\perp n} \int d(\omega - \omega_0 \phi \phi) \phi^2}{4} \]

\[ \phi \text{ turbulent diffusion of vorticity} \]

\[ \text{stronger as } V_E \rightarrow \omega/\nu \text{ from } V_E \rightarrow 0. \]

\[ \text{collisional vs. collisionless damping} \]

\[ \text{does not require instability}. \]
Coincidences on Power?

Why relation
\[
P_{\text{Tot}} \text{ LOI-SOC} \approx \left\{ \begin{array}{l}
P_{\text{HF corr. min}} \\
\text{Hugues et al.}
\end{array} \right\}
\]

Both related to collisional electron-ion coupling!

\[\nu_e \text{ LOI-SOC:}\]

\[\nu_e \sim n V_e (T_e - T_i) \sim N^2\]

\[\Delta E \text{ Electrons} = P_{\text{HF}} - \text{Transport due EDW} - \frac{P}{\Delta c} \sim \frac{1}{N}\]

\[\Delta E \text{ Ions} = \frac{P}{\Delta c} - \text{Transport due ITG (with threshold)}\]

coupling \sim \frac{1}{N^2} \Rightarrow \text{cons/ITG excited \& \overline{\nu} rises!}\]

\[\Delta E \text{ Total} = P_{\text{HF}} - \text{Transport} - \text{Transport EDW}\]

\[\text{ITG sustains LOI regime.}\]
and \( L \rightarrow H \) (Ryter)

\[ P_{\text{part}} \text{ as } n_f \text{ at } N_{\text{min}} \]

- Generally decreasing Point part of the curve is electron heated (\( \text{Cott, Eott} \))

- need \( \frac{\partial P}{\partial X} \) for \( \langle E_n \rangle \) for barrier

\[ R_{\text{el}} \sim N^2 \]

\[ \text{e} + \text{Electron} = P_{\text{el}} - P_{\text{el}} - \text{Transport} \]

\[ \text{e} + \text{Ions} = P_{\text{el}} - P_{\text{el}} - \text{Transport} \text{ (Bifurcative)} \]

\[ \Rightarrow Q = -\left[ \frac{C}{T + \varepsilon V_E^2} \right] \cdot T = 2 \varepsilon_0 \mathcal{O} \]

need sufficient power coupling to click.

\[ \text{Exact / Precise correspondence is} \]

\[ \text{TBC.} \]

\[ \Rightarrow \text{important question!} \]
$Q = -\frac{\chi_0 \alpha T}{\left[ 1 + \alpha \left( \frac{V^2}{4\omega_0^2} \right) \right]} - \chi_{\text{neo}} DT$

$\frac{\delta Q}{\delta DT} < 0 \Rightarrow \text{negative}
\text{chemical potential } \mu.$

C.F.: Hinton '96

Mahoney PD

also Hubbard in PRCF '02.
Rice: A/cstor - C Mod (again ...)

- noted bulk (evidently central) (\(4\times10^5\))
  toroidal rotation, on H-mode, with ICRF
- x-ray spectroscopy

- subsequently: Rice Scaling

\[ \Delta V_\phi \approx \Delta \frac{W}{I_p} \rightarrow \text{driven much} \]
\[ \text{cf. rotation nch} \]

\( \Delta V_\phi \rightarrow \text{increment of toroidal rotation at L=H} \)

\( \Delta W \rightarrow \text{increment of stored energy at L=H} \)

\( T_e \sim \frac{W}{P_i} \)
(r.i.e. largely pedestal formation)

\( V_p \) subsonic, but significant.

Increment = CO - current
Momentum Transport and Intrinsic Rotation

C.f. Real study of momentum transport
\begin{itemize}
\item Ida 75
\item John Rice 97
\end{itemize}\
\textit{inferred non-diffusive stress contributes to momentum balance}

\begin{itemize}
\item noted tokamak plasma rotator at significant speed w/o any external torque
\end{itemize}

\begin{align*}
\frac{d <V_p>}{dt} &= - \nabla \cdot \overrightarrow{\Gamma} \\
\overrightarrow{\Gamma} &= - \nabla P \overrightarrow{\nabla} <V_p> + \text{Something Else}
\end{align*}

\Rightarrow \text{CER\$S}
- What does it mean?

**Solomon experiment (67)**

- 3 co-beam $\rightarrow$ Period $V_p$

- 1co, 2contra-beam $\rightarrow$ Flat $V_p$

\[ \text{ie}\{ \text{plasma generates 1 beam - line of co-torque, on its own.} \} \]

\[ \text{this is serious, e.g., Fusion} \]

\[ \text{Confinement Physics} \]

- Why care for Fusion?

\[ \langle E \rangle = \langle \frac{\partial \Pi}{\partial \nabla} \rangle + \langle u \rangle \times B \]

\[ \text{Sphered, forward rotation is beneficial} \]
Table 1. Selected phenomenology of intrinsic torque

<table>
<thead>
<tr>
<th>Phenomenon</th>
<th>Signature</th>
<th>Sym. breaking</th>
<th>Key physics</th>
<th>Issue</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-mode and I-mode ETB</td>
<td>spin-up at L</td>
<td>I or H</td>
<td>(I_m) and (\nabla V_{\phi}) ↑</td>
<td>Quantitative?</td>
</tr>
<tr>
<td></td>
<td>Rice scaling (V_{\phi}(0) \sim V_T)</td>
<td></td>
<td>as (V_{\phi}(r)) ↑, (V_{\phi}) ↑</td>
<td>(V_T) or (V_{\phi})?</td>
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<td></td>
<td></td>
<td></td>
<td>and ETB forms.</td>
<td>How achieve</td>
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<td></td>
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<td></td>
<td>Cancellation experiment.</td>
<td>global cancelation?</td>
</tr>
<tr>
<td>ITB</td>
<td>(\nabla V_{\phi}) steepens</td>
<td>(V_{\phi})' in ITB</td>
<td>(\pi_{\text{rel}}) ↑ and (V_{\phi})' ↑</td>
<td>Quantitative?</td>
</tr>
<tr>
<td></td>
<td>with (V_T) in ITB</td>
<td>(V_{\phi})' in ITB</td>
<td>as (V_{\phi}(r)) ↑, (V_{\phi})' ↑</td>
<td>Relative hysteresis?</td>
</tr>
<tr>
<td></td>
<td>with (\text{vac} = 0)</td>
<td></td>
<td>Relative hysteresis of (V_{\phi})' in ITB</td>
<td>Role in de-stiffening?</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(V_{\phi}) observed</td>
<td></td>
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<tr>
<td>OH inversions</td>
<td>Inversion of (\nabla V_{\phi})'</td>
<td>Open question</td>
<td>(\nu_{\phi}) ↑ without observable change</td>
<td>Symmetry breaker?</td>
</tr>
<tr>
<td></td>
<td>around pivot for (\nu_r)</td>
<td>(r, \nu(r))' in (n,) (T) profiles.</td>
<td>in (n, T) profiles.</td>
<td>Extended flip versus localized flip</td>
</tr>
<tr>
<td></td>
<td>(v_n &gt; v_{\text{vac}}). Hysteresis in (n,) (T)-profiles.</td>
<td>(\nu_{\phi})' flip at TEM ↔ ITG transition.</td>
<td>(\nu_{\phi})' flip at TEM ↔ ITG transition.</td>
<td>Interplay with bndry</td>
</tr>
<tr>
<td>Co-NBI H-mode +ECH</td>
<td>ECH + co-NBI</td>
<td>Open question</td>
<td>(\nu_{\phi})' ↑ in (n, T)-profiles.</td>
<td>Density profile</td>
</tr>
<tr>
<td></td>
<td>(\nabla V_{\phi}) ↑</td>
<td>(r, \nu(r))' in (n, T)-profiles.</td>
<td>(\nu_{\phi})' ↑ in (n, T)-profiles.</td>
<td>+spreading</td>
</tr>
<tr>
<td>LSN ↔ USN L-mode Inversions</td>
<td>LSN ↔ USN</td>
<td>SOL flow direction</td>
<td>Change in competition between (B) and (E) field shear</td>
<td>Boundary flow</td>
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<td></td>
<td>(B) asymmetry in (L)</td>
<td>or eddy tilt</td>
<td>in USL versus USN. Core</td>
<td>+spreading</td>
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<tr>
<td></td>
<td>in (L)</td>
<td>due combination</td>
<td>responds to bndry.</td>
<td>&quot;Tail + dog&quot; problem</td>
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<td>magnetic and electric</td>
<td>+SOL flows</td>
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<td></td>
<td></td>
<td>field shear</td>
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</table>

Figure 1. 'Cancellation' experiment of Solomon et al from DIII-D [21]. A mix of 1 co and 2 counter beams yield a flat rotation profile with \(\nabla V_{\phi}\) ≃ 0. This shows that the intrinsic torque for these parameters is approximately that of a neutral beam, in the co-current direction.

Regarding the phenomenology of intrinsic torque, an interesting subset we discuss here is: (a) H-mode edge transport barrier, (b) ETB, (c) OH-reversal, (d) CON beam injection (NBI) H-mode + ECH, (e) LSN ↔ USN L-mode rotation. This discussion and that of section 5 are summarized in table 1. Of course, the classic example of intrinsic torque and intrinsic rotation is the H-mode electron transport barrier (ETB) [3]. In the absence of external torque, a spin-up is initiated at the L→H transition and builds inwards [3]. The basic trend is described by the Rice scaling \(\Delta V_{\phi}(0) \sim W/\rho\) where \(W\) is energy content and \(\rho\) refers to the change across the L→1 or L→H transition. The existence and location of the intrinsic torque have been rather convincingly established by the 'cancellation' experiment by Solomon et al [21]. The idea here was to exploit the asymmetry between co and counter-NBI H-modes due to the presence of a (hypothetical) 'intrinsic torque' \(\tau_{\text{int}}\). The result, shown in figure 1, is striking: a net counter-torque H-mode yields a rotation profile, which is flat (and zero) within the error bars! The implication is clear: the on-axis counter-NBI torque is exactly cancelled by a co-intrinsic pedestal torque! This result strongly argues for the viability of the intrinsic torque concept. It also suggests that intrinsic torque can give the appearance of a non-local intrinsic torque phenomenon, in that the intrinsic torque, situated in the pedestal, acts to flatten \(\nabla V_{\phi}\) in the core. To characterize the pedestal intrinsic torque, data base studies from Alcator C-Mod [8] indicate that central rotation in H-mode and I-mode tracks pedestal \(V_T\), i.e. \(V_{\phi}(0) \sim V_T\) ped., suggesting that the pedestal intrinsic torque is \(V_T\)-driven. Intrinsic rotation in ITBs [22–25] has received far less attention than intrinsic rotation in ETBs. This is due in part to the fact that ITBs are usually formed in plasmas subject to external torque. However, since the interaction of external and intrinsic torques is important in low torque scenarios planned for ITER, intrinsic rotation in ETBs and 'de-stiffened' states should receive more attention. Here, a de-stiffened state is one with a stronger response of the temperature gradient to heat flux increments than that exhibited by a stiff state. De-stiffening can be achieved by enhanced \(E \times B\) shear, for example. One recent experiment [10] obtained the scaling relation \(\nabla V_{\phi} \sim V_T\) for intrinsic rotation gradients in ITBs. This is reminiscent of the similar result for ETBs and again suggests that the intrinsic rotation is temperature gradient driven, as in a hot engine. To look beyond correlation to causality, that study investigated relative hysteresis between \(V_{\phi}\) and \(V_T\). Results indicated that hysteresis in \(V_{\phi}\) was stronger than in \(V_T\), possibly due to the low residual Prandtl number (i.e. \(P_{\text{resat}} \sim X_0/\chi_0\) in the ITB. Here, \(X_0\) and \(\chi_0\) are the true, not effective, diffusivities) in the ITB. Since hysteresis of a transport barrier is a consequence of the disparity between transport in the normal and the barrier state, the fact that \(\chi_0 > X_0\) in the ITB implies that hysteresis will be stronger in \(\nabla V_{\phi}\) than in \(V_T\). Recall \(\chi_0 \sim X_0\) in L-mode. A particularly compelling case for the need to consider intrinsic torque physics is the fascinating phenomenon of rotation reversals in OH or L-mode plasmas. Reversals refer to events in which the global rotation profile spontaneously reverses direction. First studied in detail in TCV [26]
and Resistive Wall Mode (kink)

\[
\hat{B} \rightarrow \text{well, finite resistivity,}
\]

Rotation of plasma relative to well is good \& stability

esp. pedestal torque,

Intrinsic rotation provides that rotation, w/o NBI (dubious for ITER).

Some other important points:

- heating method, irrelevant.
- settled confusion = RF waves
- orbit loss, ---

Classico: Hatchcoach, Rice et al. 2002

\[
\rho_{oh} = \rho_{RF} \rightarrow \text{some } (\Delta V)(\rho)
\]

\(\text{some story}\)
- Cancellation experiments ✓
- Bertalan, Duval '06, '08
- Ohmic Reversals
- Rico et al. 2011, '12
- Intrinsic rotation is universal, though strength varies
- Intrinsic rotation occurs in O unnoticed
  (L-mode).

But
- Intrinsic rotation changes spontaneously

seen both co penta ctr → edge activity
$\Rightarrow$ Ohmic reversal

Reversal occurs for $N \approx 5/4$

L0C → 50C
and if \( \text{LOC} \rightarrow \text{SOC} \rightarrow \text{IOC} \) (Aug.
Angioni, MCDermott)

- Flips back

- Points at intrinsic rotation related to type of turbulence

\( \text{CTEM} \text{ vs \ ITG } \) [P.D., '08
P.0 4/2

Ohmic reversal phenomenon nails down to connection of intrinsic rotation to turbulence, transport physics.

see also: Reams + Ech - cost of thousands.

C-F Hysteresis loop - \( \langle V_d \rangle \text{ vs } \langle V \rangle \).

- Beyond global/engineering scaling:

\( \langle V_d \rangle \text{ vs } v, v_T, \text{ and } \zeta \text{ vs } v_T, v_0 \text{ (Rice, Volonteri).} \)
Figure 2. Density ramp hysteresis loop for reversals on Alcator C-Mod at

and C-Mod [27-29], reversals are spontaneous 'flips' in the
toroidal rotation profile from co to counter (in C-Mod) which
occur as \( n \) increases and exceeds \( n_{\text{sat}} \), the density at which
confinement transitions from the linear ohmic confinement (LOC)
to saturated ohmic confinement (SOC) regime. During the
reversal, the rotation profile effectively pivots around a
fixed point inside \( q \leq 3/2 \). Interestingly, up-down density ramps
reveal back flips, but with some hysteresis, i.e. the
velocity versus density plot is a closed loop enclosing finite
area, not a straight line, as shown in figure 2. In some cases,
a rotation 'spike' (i.e. a transient, spatially localized bump
in the toroidal rotation velocity profile) was observed near
the edge just after the reversal [28]. Also, experiments on
TCV do indicate some differences between reversals in limited
and diverted discharges [30], suggesting that the effective
boundary conditions play a role in reversal dynamics. Spikes
are particularly interesting, as they may hold a clue to the
global momentum balance and rotation profile dynamics. This
is because spikes may reveal the dynamics of momentum ejection
events which help understand how the total momentum balance
of the core plasma is maintained. Building on the long-
standing idea that the evolution from LOC to SOC regime
is due to a transition from trapped electron mode (TEM)
transport to ion temperature gradient (ITG) transport excited
by collisional coupling, a speculation has arisen that inversions
are a consequence of a change in the sign of \( \Pi_{\text{T}} \) as \( n > n_{\text{sat}} \) or more generally \( n > n_{\text{ref}} \)
[31, 32]. This change reflects the dependence of \( \Pi_{\text{T}} \) on \( n \),
the group velocity of the underlying microinstability.

Alcator C-Mod has pursued fluctuation studies, the results of which are consistent with the
expected change in mode populations, but are not conclusive.

Further work is needed.

A somewhat related phenomenon, related to the effect
of ECH on co-NBI H-mode profiles, has been observed in
JT-60U [33], AUG [34], DIII-D [35], KSTAR [36] and HL-2A
[37]. Results indicate that ECH of NBI-driven H-modes tends
to flatten the otherwise peaked velocity profile, and reduce
central rotation speeds (\( \Delta V/V \sim -20\% \), in KSTAR), while
\( V_{\text{Te}} \) steepens. Profile studies indicate \( V_{\text{Te}} \sim V_{\text{Te}} \), here,
suggestive of a TEM counter-torque in the core. Correlation
of \( V_{\text{Te}} \) and \( V_{\text{Te}} \) is also indicated [38]. The H-mode pedestal
rotation profile is unchanged by ECH, suggesting that the
torque balance here is: co-NBI + pedestal co-intrinsic versus
core counter torque related to ECH. KSTAR profiles with NBI
and NBI + ECH are shown in figure 3. The data suggest
a similar paradigm to that for the OH inversion, namely a
change in the direction of the core intrinsic torque from co
to counter, due to a flip in mode propagation direction from
\( \mu_{\text{ref}} \) to \( \mu_{\text{ref}} \) as ITG gives way to TEM. Comparative gyrokinetic
stability analysis of NBI+ECH and NBI H-modes is, however,
somewhat incomplete. This follows from the sensitivity of the
results to density profile structure near the pivot radius, and
from uncertainty concerning the spatial extent of the region
where the mode population flips (according to purely linear
analysis). Fluctuation measurements are not yet available.

See [34, 36] for more details.

The importance of the edge in intrinsic rotation physics
should already be apparent. A classic example of this is the
LSN USN jog experiments of LabBombard in C-Mod L-mode
plasmas [39]. Here, 'jog' refers to the process of swing the null
point from lower (LSN) to upper (USN) positions by controlled
variation of the magnetic configuration. These are often
described as a 'tail-wags-the-dog' phenomena, since changes
from LSN to USN reverses not only scrape-off layer (SOL)
flows, but also the direction of the core rotation. Interestingly,
the effect on core rotation vanishes in H-mode, suggesting
that the tail is 'cut-off' by the sheared flow in the ETB. The
dynamics of this fascinating phenomenon are not understood.

In particular, the issue of just how flow changes penetrate
from the SOL and boundary to the core remains open. Note that this
issue may be related to the long standing mystery concerning the
\( \nabla B \)-drift asymmetry in the L \( \rightarrow \) H power threshold [40].
It is important to note here that at least two types of boundary
effects are possible. One is due to SOL flows, produced by
up-down SOL asymmetry (i.e. LSN versus USN) and driven
by in-cut asymmetry of edge particle transport [39]. The other
is due to edge stresses, induced by eddy tilting [41].

3. Towards a fundamental theory: intrinsic rotation
as the consequence of a heat engine

Recent work [7, 8] has developed a quite general theory
of intrinsic rotation as the output of a heat engine, which
exploits a heat flux-driven temperature differential (i.e. locally,
a temperature gradient \( \nabla T \)) to drive turbulence in a bounded
domain. Magnetic geometry and boundary effects break
symmetry and total momentum conservation, so that a net
toroidal flow develops. Two heat engines, a car and a tokamak,
are compared in table 2. The engine process effectively
converts radial inhomogeneity into parallel flow via symmetry-
breaking induced non-diffusive component of the Reynolds
stress \( \langle \delta u \delta v \rangle \), as shown in figure 4. The heat engine paradigm
was developed to explain the formation of geophysical flows
[42] and the solar differential rotation [43] (table 3). Both
are prime examples of flows produced by heat flux-driven
turbulence.

Here, we summarize the heat engine model, derived from
the consideration of fluctuation entropy balance. This
→ Life before intrinsic rotation?
   (Pre-History)

- CERS required for \( \langle T_i \rangle \), \( \langle V_i \rangle \) profiles

- First study - S. Scott et al., '88 TFTR

\[
\frac{\partial \chi(\mu)}{\partial t} = \text{Text} - D \cdot \frac{\partial}{\partial \mu}
\]

\[
\frac{\partial \mu}{\partial t} = -\chi(\mu) \frac{\partial \mu}{\partial \mu}
\]

- Found \( \chi_0 \sim \chi_c \), as would expect from ITG + RVE (recall!)

- Repeated ad nauseam...

- Theoretically, for ITG family turbulence, expect

\[
\sim \chi_0 \approx \chi_c
\]

but some studies showed significant deviation in reality.
\[ \Pi = - \int \chi_\phi \nabla \phi \cdot \nabla \phi \, dV + \text{something else} \]

- Two related:
  - what is something else?

naive bet: \[ \text{Ponch} \quad \text{(like particle transport)} \]

\[ \Pi = - \int \chi_\phi \nabla \phi \cdot \nabla \phi \, dV + \phi \nabla \phi \quad U < 0 \quad \text{momentum Ponch (??)} \]

How study? off axis modulated NBI
(Makoto Yoshida et al.,)
JT-60 U '08

\[ V < 0 \quad ? \]

NBI

\[ \bullet \text{but peak profile} \]

\[ \text{beam} \]
For theory of momentum pinch, see Hahn, et al., Poelers et al., '07, but it's not really/only pinch...  

- Tail flow \( \rightarrow \) Dog? \( \rightarrow \) B.L. Bombard by (Grod)

\( L \)-mode:

\( \chi \) \( \text{vs.} \) \( \chi \) \( \rightarrow \) SOL Flow reverses and Core entropic rotation reversal

c.f. Jay experiments

but LSN \( \text{vs} \) USN no effect in \( H \)-mode

\( \rightarrow \) ETB cuts the tail off \( \chi \)

TBC.

Message: The boundary is dynamic, The B.C. is non-trivial. Not simple "no-slip".
So what is the "something else"?

Residual Stress → engine

C.f. "Solar Differential Rotation" - G. Rudiger

C.f. how convection generates solar differential rotation

C.f. engine

Fusion → Fluid Turbulence → Differential Rotation

mean field hydro. → Axial Dynamo Theory

Symmetry breaking

Sum → Cylinder, cam, wheel

Q → Turbulence

\[ \langle \mathbf{u} \mathbf{u} \rangle = - \Theta \frac{\partial \mathbf{u}}{\partial t} \]
\[ \frac{\partial \langle \mathbf{v} \rangle}{\partial t} = - \nabla \cdot \mathbf{\Pi} + \ldots \]

\[ \mathbf{\Pi} = \chi \rho \frac{\partial \langle \mathbf{v} \rangle}{\partial \mathbf{v}} + \nabla \langle \mathbf{v} \cdot \mathbf{v} \rangle + \mathbf{\Pi}_{\text{resid}} \]

\[ \mathbf{\Pi}_{\text{resid}} = \mathbf{\Pi}_{\text{non-diffusive}} + \mathbf{\Pi}_{\text{stress}} \]

\[ \mathbf{\Pi}_{\text{stress}} = \mathbf{\nabla} \cdot \mathbf{\sigma} \]

\[ \mathbf{\sigma} = -\rho \mathbf{V} \mathbf{V} \]

\[ - \mathbf{D} \mathbf{P} \]

N.B. For \( \langle \mathbf{v} \rangle \), \( \frac{\partial \langle \mathbf{v} \rangle}{\partial \mathbf{v}} \quad \rightarrow 0 \)

\[ \left. \text{boundary} \right|_{\text{boundary}} \]

\[ \int_{\partial \Omega} \mathbf{\sigma} \mathbf{v} = -\mathbf{\Pi}_{\text{resid}} \]

\[ \mathbf{\Pi}_{\text{resid}} \rightarrow 0 \]

Requisite to accelerate plasma from react.
What is $\Pi_{\text{resid}}$?

c.e. neglecting pinch (delicate)

$\Pi_{\text{resid}} = \langle \tilde{U}_r \tilde{U}_n \rangle$

Reynolds stress

Key: Parallel Reynolds stress
Parallel counterpart $\langle \tilde{U}_r \tilde{U}_n \rangle$

How calculated?

\[ \frac{\partial}{\partial t} \tilde{U}_n + \frac{\partial}{\partial n} \tilde{P}_n = -\tilde{U}_n \frac{\partial \tilde{U}_n}{\partial n} - \frac{\partial}{\partial r} \left[ \frac{\partial \tilde{P}_n}{\partial r} \right] + \cdots \]

\[ \frac{\partial}{\partial t} \tilde{P}_n + \frac{\partial}{\partial n} \tilde{P}_n = -\tilde{U}_n \frac{\partial \tilde{P}_n}{\partial n} = \tilde{A}_n \tilde{U}_n \tilde{U}_n \]

Drives $\Pi_{\text{resid}}$

$\Pi_{\text{resid}} \propto \frac{\partial^2}{\partial r^2} \left( \frac{\partial \tilde{P}_n}{\partial r} \right)$

Ion pressure force $\propto$ gradient $\propto$ drive intrinsic rotation $\propto$ temperature
\[
\langle \bar{V}_r \bar{V}_n \rangle = - \chi \frac{\partial}{\partial \theta} \ln \phi \left\{ \varphi \left( \frac{\partial}{\partial \phi} \ln \phi \right) \right\} - \frac{1}{2} \ln |\phi| \right)^2 \sum \phi \text{ odd spectral moment.}
\]

Generic thermo dynamic engine.

\( \Rightarrow \) requires symmetry breaking!

c.e. select direction \( \hat{\mu} \)

analogy: \( \chi \) - effect in dynmaic theory

c.e. net breaking of reflection symmetry.

Convert radial inhomogeneity to parallel flow

\( \Rightarrow \) how?

- spectral shift
- mean \( E \times B \) shear

\( \tilting \) eddy pil,

\( E \times B \) shear
Table 4. Physics of symmetry breaking mechanisms.

<table>
<thead>
<tr>
<th>Relevant stress and mechanism</th>
<th>Spatial structure</th>
<th>Key physics</th>
<th>Macro implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\tilde{\eta}, \tilde{\eta}_1), ((\tilde{\eta}_1))^2) (Electric field shear)</td>
<td>(k_\parallel) from spectrum shift (config.) or eddy tilt (ballooning)</td>
<td>Centroid shift induces mean ((k_\parallel)) from parallel acoustic wave asymmetry</td>
<td>(\pi_{\text{sym}} \sim</td>
</tr>
<tr>
<td>(\langle \tilde{\eta}, \tilde{\eta}_1 \rangle, I') (Intensity gradient) ((I \equiv \text{intensity}))</td>
<td>(k_\parallel) from spectra dispersion due (I')</td>
<td>Spectral dispersion from intensity gradient. Linked to (\perp) Reyn. stress, also</td>
<td>(\pi_{\text{sym}} \sim I'), relevant to barriers but also for more general inhomogeneity. Can change with mode change. Ultimately tied to temp, profile curv.</td>
</tr>
</tbody>
</table>

Stress from polarization acceleration \((\tilde{E}_r \nabla^2_\perp \phi)\)

\((k_\parallel k_\perp \phi_{\parallel})^2\) stress due radial + parallel propagation, \(\perp\) tilting

Guiding centre stress from acceleration due polarization charge \((k_\parallel k_\perp \phi_{\parallel}) \neq 0\) needed

As yet unclear. Merits further study. Linked to mode radial group velocity \(v_\parallel\) and can flip direction

\(J \times B\) torque originating from polarization flux \(I' \neq 0, (k_\parallel k_\perp \phi_{\parallel}) \neq 0\) needed

\(\sim\) universal mechanism, closely related to \(ZF\) tied to \(I'\) and \(k_\parallel\) structure. Flips with \(v_\parallel\). Merits more study.

Figure 7. Symmetry breaking by \((V_\phi)\): induced spectral shift [53]. Finite \((V_\phi)\) renders the spectral centroid non-zero, and so yields \((k_\parallel)\).

is necessarily proportional to \((V_\phi)\), and cannot be so large that the underlying shear turns the underlying instability off. The correspondence between the configuration and the ballooning space manifestations of shear flow induced symmetry breaking is shown in figure 8. Note the connection between mean \(k_\parallel\) (i.e. \((k_\parallel)\)) and net eddy tilt. Clearly the real space and ballooning space approaches are equivalent.

A second, equally important mechanism for symmetry breaking is \((k_\parallel k_\perp)\) due to spatial spectral dispersion, with finite intensity gradient \(I'\) [54, 55]. This mechanism does not require a spectral shift. Rather, the requisite asymmetry is produced by the spatial profile of intensity. The origin of this

\[
\langle k_\parallel k_\perp \phi_{\parallel}^2 \rangle \approx k_\parallel^2 (r - r_0) \left( \frac{\nabla^2 \phi}{L_s} + (r - r_0) \frac{\partial}{\partial r} \phi_{\parallel} (r_0) \right)^2 + \cdots \right) \approx k_\parallel^2 (r - r_0)^2 \frac{\partial}{\partial r} \phi_{\parallel} (r_0) \frac{\partial}{\partial r} \phi_{\parallel} (r_0).
\]

Figure 9 gives an instructive heuristic sketch related to this mechanism. Note that intensity gradients will surely be steep at the boundary between regions with different confinement properties (for example, at the 'corners', which bound transport barriers where profile curvature is large). Thus, strong intensity gradients will occur near regions with large changes in \((V_\phi)\). However, one can expect an intensity gradient in

Figure 8. Shifted spectrum in real space and net eddy tilt in ballooning space. Note a Fourier transform directly relates the 'tilted' spectrum in ballooning space to the shifted spectrum in configuration space.
- Spectral intensity gradient

- N.R. net zero flux

\[ \frac{\Delta <\nu_d>}{\Delta r} = \frac{d\rho}{dr} \cdot \chi_d \]

\[ d\rho \sim <UE> \]

so on barrier intensity but

\[ \frac{\Delta <\nu_d>}{\Delta r} \]

(\( \chi_d \) not negligible)

- Reversals?

Change in mode population can force change in residual stress

c.f. McDevitt, P.O.; N. Cead; and many.