Physics 218c

Lecture 6: Confinement Transitions, especially Predator-Prey System and Limit Transition

a.) Predator-Prey and Drift Wave-Zonal Flow System.

Recall derived the coupled equations for shear flow and turbulence:

\[ \frac{d\langle N \rangle}{dt} = \frac{\partial}{\partial \vec{r}} \frac{\partial}{\partial \vec{r}} \langle N \rangle = \gamma \langle N \rangle - \frac{\nu}{N_0} \langle N \rangle^2 \]

or equivalently, in terms of energy:

\[ \frac{d\langle E \rangle}{dt} = \gamma \langle E \rangle - \frac{\nu}{N_0} \langle E \rangle^2 \]

\[ \frac{d\langle \overline{U} \cdot \overline{U} \rangle}{dt} = \langle \nabla^2 (\nabla \cdot \overline{U}) \overline{U} \rangle - \gamma \langle \overline{U} \cdot \overline{U} \rangle - \nu \langle \overline{U} \cdot \overline{U} \rangle^2 \]

\[ \frac{d\langle \overline{E} \rangle}{dt} = -\frac{\nu}{N_0} \langle \overline{E} \rangle^2 \]

\[ \langle \overline{E} \rangle = \frac{1}{2} \langle \overline{U} \cdot \overline{U} \rangle \]
and, from Reynolds stress:

\[ \partial_t \left[ \frac{2}{3} \right] l_x^2 = R_z \left[ \frac{\partial u_x}{\partial x} \right] u_x^2 - \partial_x \left[ \frac{2}{3} \right] l_x^2 \]

\[ \text{ZF growth} \quad \text{ZF drag} \]

\[ \langle l_x^2 \rangle = + \langle \nabla^2 \rangle \]

Energy conservation is straightforward!

Show this:

\[ \partial_x \left[ \int d^3 k \omega \langle \mathcal{E} \rangle + \sum_{\nu} \frac{\pi}{2} \delta \langle \mathcal{E} \rangle \right] = 0 \]

akin to energetics in QHT.

DW and particles

ZF and "waves"/"Fields"

Coupled system for DW spectrum and ZF spectral intensity
Feedback Loops I

- Closing the loop of shearing and Reynolds work
- Spectral ‘Predator-Prey’ equations

Prey $\rightarrow$ Drift waves, $\langle N \rangle$
\[
\frac{\partial}{\partial t} \langle N \rangle + D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_k \langle N \rangle - \frac{\Delta \omega_k}{N_0} \langle N \rangle^2
\]

Predator $\rightarrow$ Zonal flow, $|\phi_q|^2$
\[
\frac{\partial}{\partial t} |\phi_q|^2 = \Gamma_q \left[ \frac{\partial \langle N \rangle}{\partial k_r} \right] |\phi_q|^2 - \gamma_d |\phi_q|^2 - \gamma_N \left[ |\phi_q|^2 \right] |\phi_q|^2
\]
Some FAQ's

- What is Geometry?
  - FLR \rightarrow \rho_0 \Omega_0^2 \Phi^2

- What acts \( J_{pol} \)?

- Drifts + Particle Trapping

Length scale: drift velocity

\[ Dr \sim V_0 \Gamma \]  \( \rightarrow \) bounce time

\[ \sim \frac{\rho_i V_{th} R_e}{R} \frac{R}{V_{th} V_e} \]  

\[ \sim \frac{\rho_i}{\rho_{pol}} \]  \( \rightarrow \) poloidal gyro-radius

i.e. \( \rho_{pol} \) sets polarisation screening length

\( \rho \rightarrow \rho' \)  

\( \rho_i \rightarrow \rho_i' \)  \( \sim \) enhanced screening length and accuracy
For full analysis, see Rosenbluth and Horton.

What seeds/trigger ZF?

Answer: Nonlinear noise.

We can write:

\[ \frac{\partial}{\partial t} \langle \psi^2 \rangle \sim - \langle \mathbf{v} \cdot \nabla \psi^2 \rangle \]

\[ \mathbf{v}_i \cdot \nabla \langle \psi^2 \rangle = - \nabla \cdot \mathbf{u} \text{ etc.} \]

Then treat as Langevin equation with \( \mathbf{L} \) set by coherence times of stochastically driven ZF field.

Then

\[ \frac{\partial}{\partial t} \langle \psi^2 \rangle \sim \sum_{\ell} \left( \mathbf{v} \cdot \nabla \psi^2 \right)_\ell + \frac{1}{2} \sum_{\ell, \ell'} \mathbf{v}_\ell \cdot \nabla \psi^2 \psi^2 \]
\[ \langle D^2 \rangle \sim D^2 \rightarrow \text{grow linearly} \]

and can add noise term to zonal mode growth

\[ \rightarrow \text{can drive flow, absent modulational instability} \]

\[ \rightarrow \text{also seeds zonal density} \]

See R. Singh, P.D., APCF '12, for a complete analysis.

Zonal noise is the answer to the question of "What triggers the trigger?"

\[ \rightarrow \text{What limits ZF? / Zonal Mode?} \]

Tertiary instability
i.e. \( D W \rightarrow \text{Zonal} \rightarrow \text{Tertiary} \leftarrow \text{Instability} \)

Coupled,
primary - secondary - tertiary

\[ \text{i.e. } D W \rightarrow D V \rightarrow \text{KdV type} \]

\[ \frac{D V}{D t} > \text{Drift wave} \]

\[ \frac{D T}{D z} \]

Tertiary controversial, especially in magnetically sheared systems.

Other \( \rightarrow \) turbulent viscosity (Li, PD)

Key question: How translate into effective NL ZF damping for coupled spectra - flow system?

\[ \rightarrow \text{orgainy} \]

\[ \rightarrow \text{Staircase} \rightarrow \text{1D spatial pattern} \]

\[ \text{steps + shear layer pattern} \]

Shears \( \uparrow \) \( \uparrow \) \( \uparrow \) \( \uparrow \) \( \rightarrow \) \( \text{Bistable mixing} \)
Provocation: Staircase and Nonlocality
(with G. Dif-Pradalier, et. al.)

Analogy with geophysics: the ‘E × B staircase’

\[ Q = -n(r) \nabla T \Rightarrow Q = -\int n(r, r') \nabla T(r') \, dr' \]

- ‘E × B staircase’ width \( \equiv \) kernel width \( \Delta \)
- coherent, persistent, jet-like pattern
  \( \Rightarrow \) the ‘E × B staircase’
- staircase NOT related to low order rationals!

Dif-Pradalier, Phys Rev E. 2010
and many follow-ons.

Guilhem Dif-Pradalier
APS-DPP meeting, Atlanta, Nov. 2009
Provocation, cont’d

• The point:
  - fit: \( Q = -\int dr' \kappa(r, r') \nabla T(r') \) \( \kappa(r, r') \sim \frac{S^2}{(r - r')^2 + \Delta^2} \) \( \rightarrow \) some range in exponent
  - \( \Delta \gg \Delta_c \) i.e. \( \Delta \sim \text{Avalanche scale} \gg \Delta_c \sim \text{correlation scale} \)
  - Staircase ‘steps’ separated by \( \Delta \) ! \( \rightarrow \) **stochastic avalanches produce quasi-regular flow pattern!?**

N.B.

• The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)

• What IS new is the connection to stochastic avalanches, independent of geometry

• What is process of self-organization linking avalanche scale to zonal pattern step?
  
  i.e. How extend predator-prey feedback model to encompass both avalanche and zonal flow staircase? Self-consistency is crucial!
## Feedback Loops II

- Recovering the ‘dual cascade’:
  - Prey $\rightarrow <N> \sim <\Omega> \Rightarrow$ induced diffusion to high $k_r$ $\Rightarrow$ Analogous $\rightarrow$ forward potential enstrophy cascade; PV transport
  - Predator $\rightarrow |\phi_q|^2 \sim \left\langle V_{E,\theta}^2 \right\rangle$ $\Rightarrow$ growth of $n=0, m=0$ Z.F. by turbulent Reynolds work $\Rightarrow$ Analogous $\rightarrow$ inverse energy cascade

### Mean Field Predator-Prey Model
(P.D. et. al. '94, DI$^2$H '05)

\[
\begin{align*}
\frac{\partial}{\partial t} N &= \gamma N - \alpha V^2 N - \Delta \omega N^2 \\
\frac{\partial}{\partial t} V^2 &= \alpha N V^2 - \gamma_d V^2 - \gamma_{ NEVER} (V^2) V^2
\end{align*}
\]

to

### Reduced Model (00)

<table>
<thead>
<tr>
<th>State</th>
<th>No flow</th>
<th>Flow ($\alpha = 0$)</th>
<th>Flow ($\alpha_2 \neq 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ (drift wave turbulence level)</td>
<td>$\frac{\chi}{\Delta \omega}$</td>
<td>$\frac{\gamma}{\alpha}$</td>
<td>$\gamma + \alpha \gamma_{-1}$</td>
</tr>
<tr>
<td>$V^2$ (mean square flow)</td>
<td>0</td>
<td>$\frac{\gamma - \Delta \omega \gamma_{-1}}{\alpha}$</td>
<td>$\gamma + \Delta \omega \gamma_{-1}$</td>
</tr>
<tr>
<td>Drive/excitation mechanism</td>
<td>Linear growth</td>
<td>Linear growth</td>
<td>Linear growth</td>
</tr>
<tr>
<td>Regulation/inhibition mechanism</td>
<td>Self-interaction of turbulence</td>
<td>Random shearing, self-interaction</td>
<td>Random shearing, self-interaction</td>
</tr>
<tr>
<td>Branching ratio $\frac{V^2}{N}$</td>
<td>0</td>
<td>$\gamma - \Delta \omega \gamma_{-1}$</td>
<td>$\gamma + \alpha \gamma_{-1}$</td>
</tr>
<tr>
<td>Threshold (without noise)</td>
<td>$\gamma &gt; 0$</td>
<td>$\gamma &gt; \Delta \omega \gamma_{-1}$</td>
<td>$\gamma &gt; \Delta \omega \gamma_{-1}$</td>
</tr>
</tbody>
</table>
Feedback Loops II

- Early simple simulations confirmed several aspects of modulational predator-prey dynamics (L. Charlton et. al. '94)

Shear flow grows above critical point

\[ \frac{\bar{n}}{n_0} \]

\[ \langle V'_\theta \rangle \]

\[ \langle (\bar{n}/n_0)^2 \rangle^{1/2} \]

\[ \frac{\bar{\mu}}{\Omega_i} \]

'With Flow' and 'No Flow'. Scalings of \( \langle (\bar{n}/n_0)^2 \rangle \) appear. Role of damping evident

Generic picture of fluctuation scale reduction with flow shear

\[ \mu \to \infty \Rightarrow \text{Dimits Sight} \]
Progress II: β-plane MHD (with S.M. Tobias, D.W. Hughes)

Model

• Thin layer of shallow magneto fluid, i.e. solar tachocline

• β-plane MHD ~ 2D MHD + β-offset i.e. solar tachocline

\[
\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \tilde{f}
\]

\[
\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \quad \tilde{B}_0 = B_0 \hat{x}
\]

• Linear waves: Rossby – Alfvén

\[
\omega^2 + \omega \beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0 \quad (R. Hide)
\]


Progress II, cont’d

Observation re: What happens?

- Turbulence → stretch field → \( \langle \tilde{B}^2 \rangle \gg B_0^2 \) i.e. \( \langle \tilde{B}^2 \rangle / B_0^2 \sim R_m \)  
  (ala Zeldovich)

- Cascades : forward or inverse?
  - MHD or Rossby dynamics dominant !?

- PV transport: \( \frac{dQ}{dt} = -\int dA \langle \tilde{\nu} \tilde{q} \rangle \longrightarrow \) net change in charge content  
  due PV/polarization charge flux

Now \( \frac{dQ}{dt} = -\int dA \left[ \langle \tilde{\nu} \tilde{q} \rangle - \langle \tilde{B} \cdot \tilde{J} \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{\nu} \tilde{v}_x \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\} \longrightarrow \) Reynolds mis-match

  PV flux current along tilted lines \( \longrightarrow \) vanishes for Alfvenized state

Taylor: \( \langle \tilde{B} \cdot \tilde{J} \rangle = \partial_x \langle \tilde{B}_x \tilde{B}_y \rangle \)
Progress II, cont’d

- With Field

\[
\begin{align*}
B_0 &= 10^{-1} \\
B_0 &= 10^{-2} \\
B_0 &= 0 \\
B_0 &= 10^{-3}
\end{align*}
\]
Progress II, cont’d

- Control Parameters for \( \tilde{B} \) enter Z.F. dynamics
  - Like RMS, Ohm’s law regulates Z.F.

- Recall
  - \( \left< \tilde{v}^2 \right> \) vs \( \left< \tilde{B}^2 \right> \)
  - \( \left< \tilde{B}^2 \right> \sim B_0^2 R_m \rightarrow \text{origin of } B_0^2 / \eta \text{ scaling !?} \)

- Further study → differentiate between:
  - cross phase in \( \left< \tilde{v}_r \tilde{q} \right> \) and O.R. vs J.C.M
  - orientation: \( \tilde{B} \parallel \tilde{V} \) vs \( \tilde{B} \perp \tilde{V} \)
  - spectral evolution
b) \( \text{H} \rightarrow \text{He} \) Transition

Phenomenology:

\( \rightarrow \text{H} \rightarrow \text{He} \) characterized by:

\( \rightarrow \text{edge gradient steepness} \)

\( \rightarrow \text{pedestal formation} \)

\( \rightarrow \text{confinement improvement} \)

\( \rightarrow \text{fluctuation (low k) drop} \uparrow \)

High \( k \)'s persist in pedestal.

\( \rightarrow \text{Increase in ExB shear formation} \)

of \( E \) wall

\[ E \]

Learning can develop from inner edge layer. (Schmitz 2021)
Power threshold ($P_{\text{crit}}$) — critical to ITER

- recall $P_{\text{crit}} = E + \text{Sawtooth}$
- DD drift asymmetry

\[ \text{\smaller Lower } P_{\text{crit}} \quad \text{\smaller Higher } P_{\text{crit}} \]


- $P_{\text{thresh}}$ — major concern ITER
- related LOE-GO

why? $	o$ coupling to $\omega_{\text{ps}}$

c.i. $\partial P_c \sim E_r$

increased $n$ assures stronger electron-ion coupling $\sim n T V_e (T_e - T_i)$
Some evidence $P_{th} \sim nB_T$

Current ??

- isotope $D$ - lower in D than H, etc.

Relation to microphysics unclear.

- Other points:
  - universal to all heating method
  - limiter and diverted plasma, but
  - never in outside limiter
  - $P_{th}$ higher for limiter plasma.
- observed in stellarators

W7AS, TJ-II, LHD

- RFP $^? \rightarrow QSM ?!$

$P_{oh}$ big $\rightarrow$ Boundary

- Hysteresis happens

$P_{lh} > P_{hl}$.

Poorly understood $\rightarrow$ Very important.

- $P_{lh}$ in RMP plasmas $\rightarrow$

Important for ELM mitigation.
How? 

Trigger + DP Feedback

Energy to Shear Flow (ZF)

\[ \text{DP Steeper} \]

Turbulence collapses

\[ \Rightarrow \text{Multi-step} \]

\[ \Rightarrow \text{Many examples. } \text{DIII-D, EAST, TJ-II, AUG, HL-2A, TEXTOR} \)

\[ \Rightarrow \text{A few dissenters} \ldots \text{not many} \]

\[ \text{JFT-2M, AUG, HL-2A} \]

\[ \Rightarrow \text{Orbit loss?} \]

\[ \Rightarrow \text{Is there a unique route to transition?} \]
The Oscillating Flow Layer Widens Radially (Frequency Decreases) - Steady Flow after Final H-Mode Transition

A weak $E \times B$ flow layer exists in **L-mode** (L-mode shear layer)

At the **I-phase transition**, the $E \times B$ flow becomes more negative first near the separatrix, flow layer then propagates inward

The flow becomes steady at the **final H-mode transition** (after one final transient)
During the I-phase, the Mean Shear $\langle \omega_{E\times B} \rangle$ Increases with Time and Eventually Dominates

Outer layer Shearing Rate (Mean flow + ZF)

ExB Flow from DBS (Includes ZF)

Diamagnetic component of ExB flow (from ion pressure Profile)

R~2.265m

L. Schmitz TTF'11, PRL'12
Feedback Loops III

- $\nabla P$ coupling
  \[ \partial_t \varepsilon = \varepsilon N - a_1 \varepsilon^2 - a_2 V^2 \varepsilon - a_3 V_{ZF}^2 \varepsilon \]
  \[ \partial_t V_{ZF} = b_1 \frac{\varepsilon V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF} \]

\[ \varepsilon \equiv DW \text{ energy} \]
\[ V_{ZF} \equiv ZF \text{ shear} \]
\[ N \equiv \nabla \langle P \rangle \equiv \text{pressure gradient} \]

- Simplest example of 2 predator + 1 prey problem (E. Kim, P.D., 2003)

\[ \gamma = \gamma (\nabla P) \]
\[ \partial_t N = -c_1 \varepsilon N - c_2 N + Q \]
\[ V = dN^2 \text{ (radial force balance)} \]

\[ \text{i.e. prey sustains predators} \]
\[ \text{predators limit prey} \]
\[ \text{usual feedback} \]

- Multiple predators are possible

- $\nabla \langle P \rangle$ as both drive and predator

- Relevance: LH transition, ITB
  - Builds on insights from Itoh's, Hinton
  - $ZF \Rightarrow$ triggers
  - $\nabla \langle P \rangle \Rightarrow$ 'locking in'
Feedback Loops III, cont’d

- **Observations:**
  - ZF’s trigger transition, $\nabla \langle P \rangle$ and $\langle V \rangle$ lock it in
  - Period of dithering, pulsations .... during ZF, $\nabla \langle P \rangle$ oscillation as $Q \uparrow$
  - Phase between $\varepsilon$, $V_{ZF}$, $\nabla \langle P \rangle$ varies as $Q$ increases
  - $\nabla \langle P \rangle \leftrightarrow$ ZF interaction $\Rightarrow$ effect on wave form