Justify all your answers to all problems. Write clearly.

Time dilation; Length contraction: $\Delta t = \gamma \Delta t_0$; $L = L_0 / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - ut)$; $y' = y$; $t' = \gamma(t - ux / c^2)$

Velocity: $v'_x = \frac{v_x - u}{1 - uv_x / c^2}$; $v'_y = \frac{v_y}{\gamma(1 - uv_x / c^2)}$; $\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}$

Inverse transformations: $u \rightarrow -u$, primed $\leftrightarrow$ unprimed; Doppler: $f' = f \frac{1 \pm u / c}{\sqrt{1 \mp u / c}}$

Momentum: $\vec{p} = \gamma m \vec{v}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1) mc^2$

$E = \sqrt{p^2 c^2 + m^2 c^4}$; rest energy: $E_0 = mc^2$

Electron: $m_e = 0.511 \text{ MeV} / c^2$; Proton: $m_p = 938.26 \text{ MeV} / c^2$; Neutron: $m_n = 939.55 \text{ MeV} / c^2$

Atomic unit: $1 \mu = 931.5 \text{ MeV} / c^2$; electron volt: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi = hc / \lambda - \phi$; $\phi$ = work function

Stefan law: $I = \sigma T^4$, $\sigma = 5.67037 \times 10^{-8} \text{ W} / \text{ m}^2 \cdot \text{ K}^4$; Wien's law: $\lambda_m T = 2.8978 \times 10^{-3} \text{ m} \cdot \text{ K}$

$I(T) = \int_0^\infty I(\lambda, T) d\lambda$; $I = (c / 4)u$; $u(\lambda, T) = N(\lambda) E_{av} (\lambda, T)$; $N(\lambda) = \frac{8\pi}{\lambda^4}$

Boltzmann distribution: $N(E) = C e^{-E / kT}$; $N = \int_0^\infty N(E) dE$; $E_{av} = \frac{1}{N} \sum_0^\infty E N(E)$

Classical: $E_{av} = kT$; Planck: $E_n = n\epsilon = nhf$; $N = \sum_0^\infty N(E_n)$; $E_{av} = \frac{1}{N} \sum_0^\infty E N(E_n)$

Planck: $E_{av} = \frac{hc / \lambda}{e^{hc / \lambda kT} - 1}$; $hc = 1.2400 \text{ eV} \cdot \text{ nm}$; $\lambda_m T = hc / 4.96k$; $\sigma = \frac{2\pi^3 k^4}{15c^2 \hbar^3}$

Boltzmann constant: $k = (1 / 11,604) eV / K$; $1\text{Å} = 1\text{Å} = 0.1 \text{ nm}$

Compton scattering: $\lambda' - \lambda = \frac{h}{m_c} (1 - \cos \theta)$; $\frac{h}{m_c} = 0.0243 \text{ Å}$

de Broglie: $\lambda = \frac{h}{p}$; $f = \frac{E}{h}$; $\omega = 2\pi f$; $k = \frac{2\pi}{\lambda}$; $E = h\omega$; $p = \hbar k$; $E = \frac{p^2}{2m}$

Wave packets: $y(x, t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x, t) = \int dk a(k) e^{ik\Delta x}$, $\Delta k \Delta x \sim 1$; $\Delta \omega \Delta t \sim 1$

Group and phase velocity: $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$
Problem 1 (6 points)
Incident photons of wavelength 0.001 nm are Compton scattered, and the scattered photons are observed at an angle 45° relative to the incident beam.
(a) What is the wavelength of the scattered photons, in nm?
(b) What is the kinetic energy of the electrons scattered by these photons, in MeV?

Problem 2 (6 points)
An electron is moving with kinetic energy 24.3 eV. A photon incident on it scatters off it and the electron stops moving.
(a) Find the wavelength of the incident photon.
(b) Find the wavelength of the scattered photon.
Hint: use energy and momentum conservation.

Problem 3 (6 points)
An electron is confined to a region of space of length 0.5 nm.
(a) Estimate the uncertainty in its momentum using the uncertainty principle. Give your answer in units eV/c.
(b) Estimate its kinetic energy, in eV.

Problem 4 (6 points)
An electron has de Broglie wavelength 0.002 nm.
(a) Find its momentum, in units eV/c
(b) Find its kinetic energy, in eV.

Problem 5 (6 points)
An electron is described by a wavepacket of constant amplitude in a range Δk around k₀, with k₀=1 nm⁻¹ and Δk=0.1 nm⁻¹.
(a) Estimate the uncertainty in the position of this electron, in nm.
(b) Estimate the speed of this electron, in m/s.